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Determination of the effect of injection wells on production wells in their work dynamics by using hydrodynamic modeling[*]

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We consider a method for determining a mutual influence of a group of production and injection wells using a module of hydrodynamic modeling. The obtained results of this mutual influence are widely used in practice – from maintaining the reservoir pressure to an effective optimization of water flooding. To simulate the process of multiphase mixture filtration we use the finite element method and a specific technology that allows balancing flows across cell boundaries of the finite element mesh. The balancing method is based on the selection of corrective additives that satisfy the minimum functional of the balance of volumes of the filtered mixture with regularization. The model of mutual influence of wells uses the dependence of the wellhead pressure obtained as a result of hydrodynamic modeling, as well as volumes of an injected and/or taken-off mixture (with different signs). For a producing well of interest, the wellhead pressure is approximated by a function of the volumes of the injected mixture in a group of neighboring wells. Regression coefficients are calculated by the least squares method. The article also provides results of studies that consider the application of the method on three specific tasks that, in turn, emulate the operation of production and injection wells. To evaluate the degree of influence of a nearby well on a well of interest, a significance criterion is introduced in the form of ratios of residual sums of squared deviations. The calculation results indicate that the estimated effect of the wells corresponds to the physics of the process, which confirms the correctness of the developed procedure for analyzing the interaction of injection and production wells, and also indirectly confirms the correctness of the implementation of the module for direct problems solving. The creation of the hydrodynamic model, the calculations, as well as the analysis of the mutual influence of the wells is implemented interactively in an automated software-information system.

Keywords: reservoir simulation, filtration, finite element method, balancing flows, wells interaction, regression, influence, information system

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INTRODUCTION

Today improving the quality of production processes in the oil industry is challenging without the use of high-tech software that allows building a digital model of the deposit, carrying out three-dimensional modeling and making interactive analysis of the results. Software systems of this kind should be based on the use of three-dimensional mathematical models that reflect the fundamental laws of physics, in particular hydrodynamics [1, 2]. Also, a very important task is to determine the mutual influence of production and injection wells. The obtained results of mutual influence are actively used in practice, for example, for maintaining reservoir pressure, or conducting effective water flood optimization [3].

The existing specialized software systems for modeling (e.g., ECLIPSE [4], RSM [5], etc.) use the finite volume method, due to which a balanced solution is obtained (observing the law of mass balance) [6]. However, it has a number of disadvantages for modeling areas with curved boundaries. Also, most studies to determine the mutual influence of wells use methods of mathematical statistics, or some heuristic approaches. However, a formal calculation of the mixture (or oil) withdrawal values at the producing well using a group of neighboring wells is not a universal approach, even with simple model examples. Besides the use of additional data on the spatial orientation of wells and the construction of diagrams taking into account environmental parameters does not allow achieving any acceptable results [7].

1. STATEMENT OF THE PROBLEM

Since one of the main requirements for hydrodynamic modeling systems is to comply with the law of mass conservation, numerical schemes based on the finite volume method are quite common. This method is based on the balance of flows across cell boundaries of a discrete model.

At the same time the finite element method (FEM) allows us to use a much more heterogeneous object geometry which is typical to describe the reservoir and wells geometry in this class of problems. Moreover, a formal application of FEM in the Galerkin formulation does not ensure compliance with the mass conservation law [8–10]. Therefore, modifications are most commonly used for practical applications [11]. However, this approach entails an enormous computational cost due to the use of very detailed spatial meshes to obtain solution convergence.

In this paper we will describe an approach to the use of the finite element method, which guarantees the implementation of conservation laws and does not impose additional requirements on the spatial mesh fineness. Also, a description will be given of a method based on this approach to determine the influence of injection wells on production wells using data on the injection volume, production volume and pressure on production wells.

2. FINITE ELEMENT APPROXIMATION

The process of a multiphase flow of an incompressible fluid in the isothermal area Ω can be described by a system of differential equations [12]

$$-\operatorname{div}(\rho^m \vec{u}^m) = \frac{\partial}{\partial t}(\Phi \rho^m S^m) + \tilde{f}, \quad m = \overline{1, M}, \tag{1}$$

$$\vec{u}^m = -\mathbf{K}\frac{\kappa^m}{\eta^m}\operatorname{grad}\left(P + P_c^m\right), \quad m = \overline{1,M}, \tag{2}$$

where $\vec{u}^m$ is a filtration velocity of the m phase in porous media (Darcy's law); S^m is the saturation of the m phase; η^m is dynamic viscosity; κ^m is relative permeability; ρ^m is phase density; P is pressure; P_c^m is the capillary pressure phase; $\tilde{f}$ is the production or injection of the phase in the field; Φ is the structure of porous rocks, and K is structural permeability rocks.

With constant values of Φ, K and ρ^m equations (1) – (2) can be converted into the problem [13]

$$-\operatorname{div}\left(\sum_{m=1}^{M}\mathbf{K}\frac{\kappa^m}{\eta^m}\operatorname{grad}\left(P + P_c^m\right)\right) = 0, \tag{3}$$

with boundary conditions

$$P\big|_{\Gamma_1} = P_g, \tag{4}$$

$$\sum_{m=1}^{M}\mathbf{K}\frac{\kappa^m}{\eta^m}\frac{\partial P}{\partial n}\bigg|_{\Gamma_2} = \theta, \tag{5}$$

where Γ_1 is far lateral boundaries of the domain Ω; Γ_2 is an impermeable boundary with $\theta = 0$ and the boundaries of the wells through which the production or injection of the mixture is specified.

Problems (3) – (5) will be solved using the finite element method to discretize this Ω area to Ω_e cells. In each cell the rock and mixture parameters are constant. The pressure function P can be represented as the decomposition of linear basis functions ψ_i :

$$P = \sum_{j} p_j \psi_j. \tag{6}$$

The weight decomposition p_j can be found by solving sparse linear tems [14]

$$\mathbf{Ap} = \mathbf{b},$$

where $\mathbf{p}$ is a vector of unknown weights while components of the matrix $\mathbf{A}$ and the vector $\mathbf{b}$ of the right side are defined by the relations

$$A_{ij} = \int_{\Omega}\left(\mathbf{K}\sum_{m=1}^{M}\frac{\kappa^m}{\eta^m}\operatorname{grad}\psi_j\right)\operatorname{grad}\psi_i d\Omega,$$

$$b_i = \int\limits_{\Gamma_2} \theta \psi_i d\Omega - \int\limits_{\Omega} \left(\mathbf{K} \sum_{m=1}^{M} \frac{\kappa^m}{\eta^m} \mathrm{grad} P_c^m \right) \mathrm{grad} \psi_i d\Omega \ .$$

We make an approximation of the fluid flow across boundaries Γ_i of the cell area Ω_e. For this we calculate the filtration velocity

$$\vec{u}_{\Gamma_i} = -\mathbf{K} \sum_{m=1}^{M} \frac{\kappa^m}{\eta^m} \mathrm{grad} \left(\sum_j p_j \psi_j + P_c^m \right),$$

according to which the volumes of the filtered mixture can be calculated on the face Γ_i of the finite element Ω_e:

$$Q_{\Gamma_i} = \int\limits_{\Gamma_i} \vec{u}_{\Gamma_i} \cdot \vec{n}_{\Gamma_i} d\Gamma \ , \tag{7}$$

where $\vec{n}_{\Gamma_i}$ is a normal vector to the face Γ_i. The flow direction is defined by the value $Sg_{\Gamma_i}^{\Omega_e}$, which is equal to 1 if Q_{Γ_i} flows into the element Ω_e and is equal to -1 if Q_{Γ_i} flows out.

The flowing volume can be different on both sides of the face, and the flowing volume may not be equal to the flowing volume, due to an error in the numerical solution Therefore, we will use the balancing process flows for the averaged volume on the face $\hat{Q}_{\Gamma_i}$.

Let us find such corrective additives δQ_i (where i is the global number of faces) for each finite element, that they, on the one hand, minimize the total system unbalance, and, on the other hand, are close to the calcuated numerical flow. These corrective additives can be found as a result of δQ_i minimization following the functional [15]:

$$\sum_{e=1}^{N^e} \beta_e \left(\sum_{i \in I_{\Omega_e}} \left(Sg_{\Gamma_i}^{\Omega_e} \cdot \left| \hat{Q}_{\Gamma_i} \right| + Sg_{\Gamma_i}^{\Omega_e} \cdot \delta Q_i \right) \right)^2 + \sum_{i=1}^{N^f} \alpha_i (\delta Q_i)^2 \ , \tag{8}$$

where e is the number of the finite element; I_{Ω_e} is a set of faces Γ_i of the finite element Ω_e; N^e and N^f are numbers of finite elements and faces; β_e and α_i are parameters. The parameter values β_e should be chosen as small as possible but still they should ensure the required imbalance δQ^{crit}:

$$\sum_{e=1}^{N^e} \beta_e \left| \sum_{i \in I_{\Omega_e}} \left(Sg_{\Gamma_i}^{\Omega_e} \cdot \left| \hat{Q}_{\Gamma_i} \right| + Sg_{\Gamma_i}^{\Omega_e} \cdot \delta Q_i \right) \right| < \delta Q^{crit} \ .$$

As a result of minimizing the functional (8) to δQ_i we get a system of linear equations

$$(\mathbf{B} + \boldsymbol{\alpha})\mathbf{q} = \mathbf{d},\tag{9}$$

where $\mathbf{q}$ is a vector of unknown values δQ_i, $\boldsymbol{\alpha}$ is a matrix with elements α_i, and components of the matrix $\mathbf{B}$ and vector $\mathbf{d}$ can be found by using the relations

$$B_{ij} = \begin{cases} 2, \ i = j, \\ \beta_e \cdot Sg_{\Gamma_i}^{\Omega_e} \cdot Sg_{\Gamma_j}^{\Omega_e}, \ i,j \in I_{\Omega_e}, i \neq j, e = 1 \dots N^e, \\ 0, \ else \end{cases}$$

$$d_i = -\sum_{e=1}^{N^e} Sg_{\Gamma_i}^{\Omega_e} \cdot \sum_{j \in I_{\Omega_e}} \hat{Q}_{\Gamma_j,\Omega_e}.$$

Rows/columns corresponding to the boundary conditions (5) on the faces of the mixture volume should be deleted from the system (9).

The balanced volume mixtures $\tilde{Q}_{\Gamma_i,\Omega_e}$ are defined as

$$\tilde{Q}_{\Gamma_i,\Omega_e} = Sg_{\Gamma_i}^{\Omega_e} \cdot \left| \hat{Q}_{\Gamma_i,\Omega_e} \right| + Sg_{\Gamma_i}^{\Omega_e} \cdot \delta Q_{i,\Omega_e}.$$

Using the volumes of the mixture $\tilde{Q}_{\Gamma_i,\Omega_e}$ we can calculate the saturation distribution at the current time step. We denote by

$$\tilde{Q}_{\Gamma_i,\Omega_e}^m = \tilde{Q}_{\Gamma_i,\Omega_e} \frac{\kappa^m}{\eta^m \sum_{n=1}^{M} \kappa^n / \eta^n}$$

an instantaneous (per unit time) volume of the m phase overflowing through the face Γ_i. Then during the time Δt the volume of the m-th phase equal to

$$V_{\Gamma_i,\Omega_e}^m = \left| \tilde{Q}_{\Gamma_i,\Omega_e}^m \right| \cdot \Delta t.$$

will overflow through the face Γ_i.

At the beginning of the time step, the volume of the m phase in the cell Ω_e is

$$\bar{V}_{\Omega_e}^m = \Phi \bar{S}^m \mathrm{mes}(\Omega_e),$$

where $\bar{S}^m$ is the saturation of the m phase at the beginning of the time step, and $\mathrm{mes}(\Omega_e)$ is the volume Ω_e.

Now we can determine the m phase saturation at the end of the current time:

$$S^m = \frac{\sum\limits_{i \in I_{in,\Omega_e}} V^m_{\Gamma_i,\Omega_e} - \sum\limits_{j \in I_{out,\Omega_e}} V^m_{\Gamma_j,\Omega_e} + \bar{V}^m_{\Omega_e}}{\Phi \cdot \mathrm{mes}(\Omega_e)},$$

where I_{in,Ω_e} is a set of face numbers through which the mixture flows into Ω_e, and I_{out,Ω_e} is a set of face numbers through which the mixture flows out from Ω_e.

We will repeat the described process on each time layer.

3. DETERMINATION OF THE INFLUENCE OF WELLS

To determine an influence of injection wells on production wells it is proposed to use data on an average daily injection volume changing in time in injection wells, the production volume and pressure of producing wells.

According to the obtained solution (6) pressure on injection wells can be calculated as

$$P_k = \sum_{j \in I_{W_k}} \int_{\Gamma_j} P_{\Gamma_j} d\Gamma \bigg/ \sum_{j \in I_{W_k}} \int_{\Gamma_j} d\Gamma, \qquad (10)$$

where I_{W_k} is a set of face numbers Γ_i of the k injection well.

The pressure value (10) on the production well W_k at the time t_i can be approximated by the function $\tilde{P}_k$ of the volume Q_j of the injected mixture on a group of neighboring wells:

$$\tilde{P}_k(t_i) = a_0 + \sum_{j=1}^{m} a_j Q_j(t_i). \qquad (11)$$

The normalized residual sum of squares of deviations (RSS) can be considered as a criterion of the proximity of the dependence $\tilde{P}_k$ to the function P_k. In this case, the coefficients $a_0, a_1,..., a_m$ are found by the least squares method.

To assess the degree of influence of the j-th well let us introduce the value significance test (hereinafter – an influence coefficient) in the following form

$$\chi_j = (RSS_j - RSS)/RSS, \qquad (12)$$

where the RSS_j value is obtained by finding the $\tilde{P}_y$ function coefficients excluded from the j-th well.

The influence coefficient χ_j will be maximum for injection wells that have the greatest impact on the production well of interest.

4. SOFTWARE – INFORMATION SYSTEM

For convenient creation of a hydrodynamic model of the field, as well as for viewing the obtained calculation results, a software-information system was developed. Fig. 1 shows a graphical interface of this system.
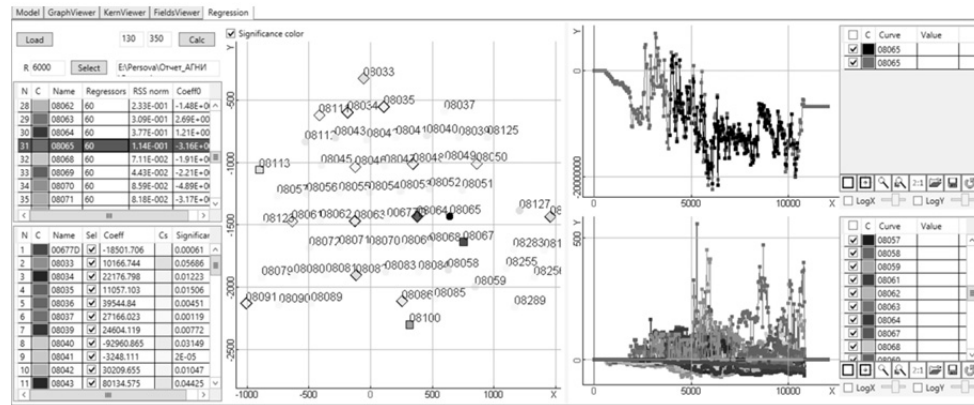

Fig. 1. Window view of the information system for determining the wells interference

Рис. 1. Вид информационной системы для определения взаимовлияния скважин

The system consists of three logical units. Elements of the input parameters for the analysis model, lists of production and injection wells with the characteristics of influence and control buttons are located in the left part of the GUI window. A graphical display of the wells in the plan (Oxy planes) in various color gradations is given in the center. A unit for displaying of characteristics of wells mutual influence is shown on the right. A circle is used to display wells in the center; a square is used to display injection wells, and a rhombus is used to display production wells which were converted into injection wells.

The list of analyzed production wells is presented in a table and contains the following information: the serial number of the well (column N), the color of the well display (C), the identifier of the well (Name), the number of selected regressors or affecting wells (Regressors), a normalized residual sum of squares (RSS norm), and a free term (Coeff0).

When you click on the production well of interest, the list of influencing injection / produc wells is updated, which is also presented in the form of a table containing the following information: the serial number of the well (column N), the color of the well display (C), the identifier (name) of the well (Name),the mark of selecting this well in the analyzed model (Sel) (the system provides an ability to include/exclude individual wells from the analysis, and also to use for analysis only a group of wells located inside a circle with a radius (R), a linear regression coefficient for the given well (Coeff), the color of the well display by the degree of its influence on the studied production well (C), the influence coefficient reflecting the degree of influence of the current well on the analyzed production well (Significance).

The information system provides an ability to determine an influence of wells on the time interval of interest. To do this, you need to set two boundaries of the interval. At the top of the right block, the graphs of the initial pressure and the pressure on the production well in the selected time interval found by the proposed method are displayed (shown in black). At the bottom of the right block are graphs of injections and productions from all wells used in the analysis. They can be viewed separately (turning on/off the corresponding graph), and it is possible to change the scale.

5. COMPUTATIONAL EXPERIMENTS

Testing of the developed module was carried out on synthetic data obtained using the module for solving direct problems.

Let us consider a three-layer model in which the first and third permeable layers are separated by impermeable interlayers (the model task was carried out using an automated system [16]). In this case, we consider three versions of this model. In the first case (model 1) a dividing interlayer is completely impermeable, but homogeneous permeability is set in the top and bottom layers (oil saturation distribution and location of wells are shown in Fig. 2, *a*, *b*) (oil saturation distribution and location of wells are shown in Fig. 2, *a*, *b*). In the second case (model 2) an impermeable "partition" is set in the upper layer (see. Fig. 2, *c*), while in the third case 3 (model 3) a permeable zone is set in the impermeable interlayer which separates the upper and lower layers (see. Fig. 2, *d*).
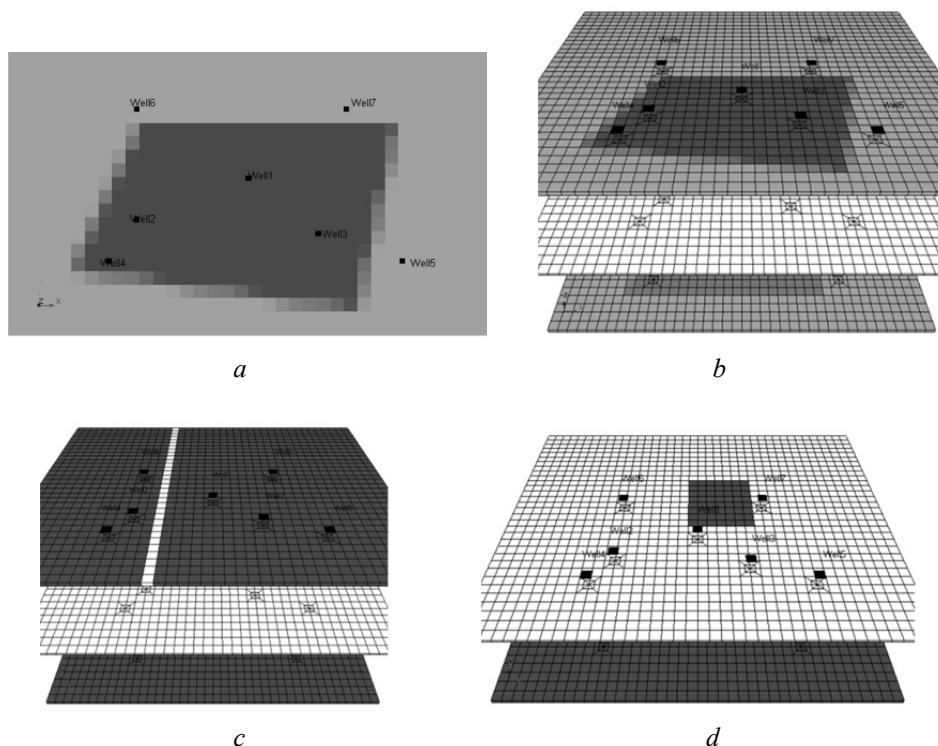

a   *b*

c   *d*

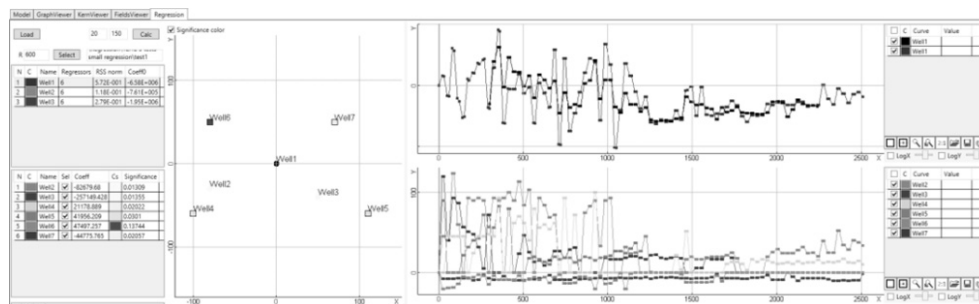*Fig. 2*. Models to test procedures for determining the mutual influence of injection and production wells:

a – the distribution of saturation in the upper layer and the location of the wells; *b* – a layered view saturation in model 1; *c* – a layered view of permeability in model 2; *d* – a layered view of permeability in model 3 (with a removed upper layer)

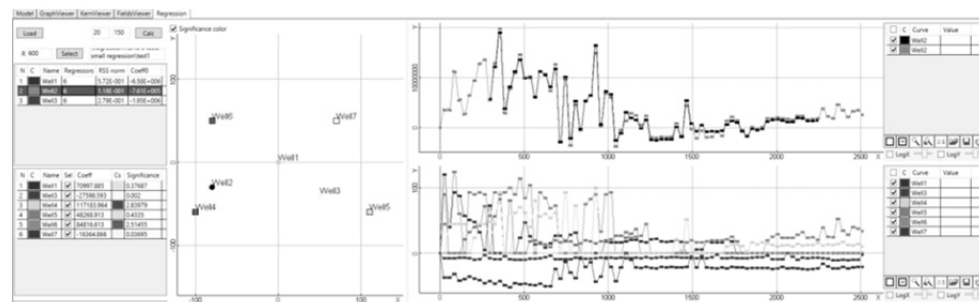*Рис. 2*. Модели для тестирования процедуры определения взаимовлияния нагнетательных и добывающих скважин:

а – распределение насыщенности в верхнем слое и расположение скважин; *б* – послойный вид насыщенности в модели 1; *в* – послойный вид проницаемости в модели 2; *г* – послойный вид проницаемости в модели 3 (со снятым верхним слоем)

Thus, Well 1 takes off only from the upper layer, Well 2 takes off from both the top and the bottom layers, and Well 3 takes off only from the bottom layer. In turn, Well 4, Well 5 and Well 6 inject into the upper layer and Well 7 injects into the lower layer.
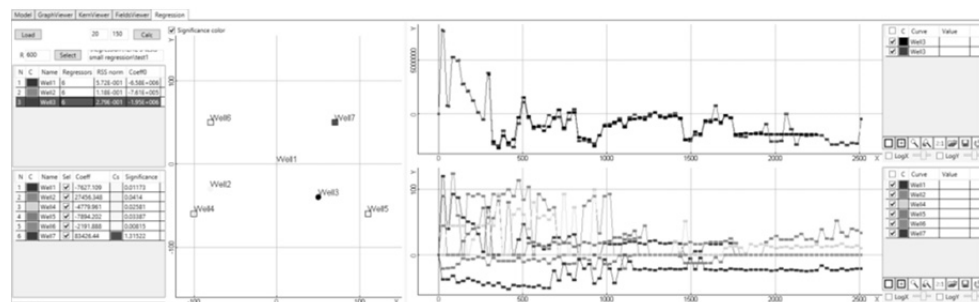
Fig. 3 presents the results of procedures to determine the influence of wells for Model 1. It can be seen that Well 1 is mainly influenced by Well 6 (as Well 7 injects into the layer separated by an impermeable interlayer and Well 4 and Well 5 are separated by production wells, namely Well 2 and Well 3). Well 2 is affected by Well 6 and Well 4 (with an influence coefficient of 2.5-2.8), and Well 3 is affected by Well 7 (which, despite the fact that it is further located, injects in the same layer from which production Well 3 takes off, and this layer is separated by impermeable interlayers).

a

b

c

Fig. 3. The influence of Well 1 (*a*), Well 2 (*b*) and Well 3 (*c*) according to the synthesized model 1

Рис. 3. Влияние на скважины Well 1 (*а*), Well 2 (*б*) и Well 3 (*в*) по данным, синтезированным для модели 1

Fig. 4, *a*, *b* shows the results for model 2. It can be seen that the influence on Well 1 dropped sharply since it became a separate impermeable "barrier", and the influence on Well 2, on the contrary, increased dramatically (the influence coefficient became 3.5). Fig. 4, *c* presents the results of model 3. It can be seen that due to the appearance of permeable zones in the interlayer Well 3 became affected by Well 5 which injects into the upper layer in addition to Well 7.
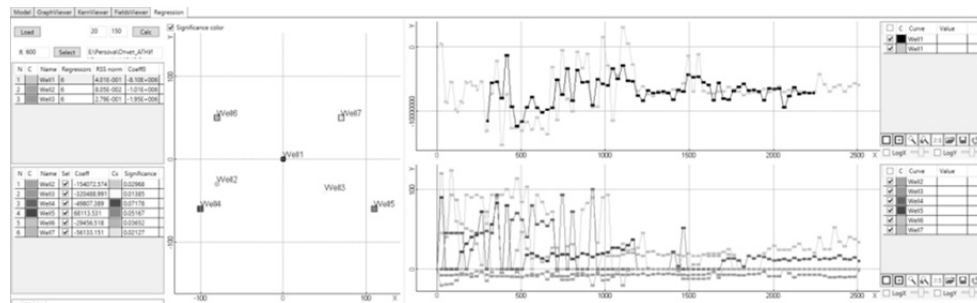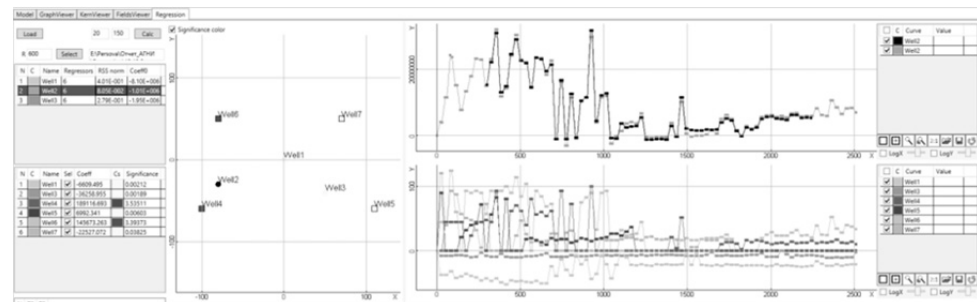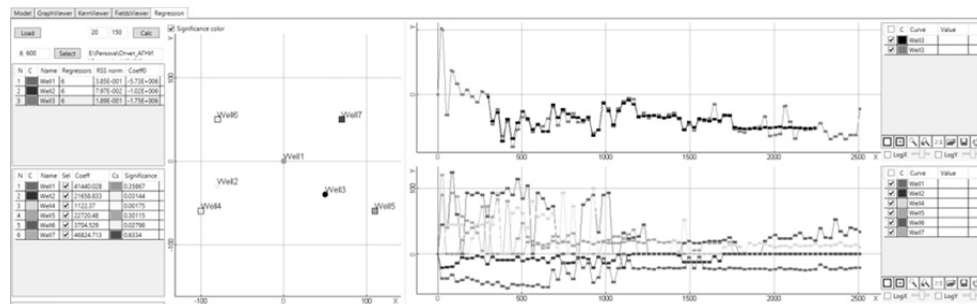

a

b

c

Fig. 4. The influence of Well 1 (*a*) and Well 2 (*b*) according to the synthesized model 2, and Well 3 (*c*) according to the synthesized model 3

Рис. 4. Влияние на скважины Well 1 (*а*) и Well 2 (*б*) по данным, синтезированным для модели 2, и на скважину Well 3 (*в*) по данным, синтезированным для модели 3

CONCLUSION

The method for determining the influence of injection wells on production wells is presented in the paper. The aforementioned method of finite-element modeling of hydrodynamic processes avoids the disadvantages of the classical Galerkin

method and guarantees the observance of the law of mass balance of the filtered components of the mixture. The conducted study of characteristic synthetic models allows us to conclude that the estimated influence of wells corresponds to the physics of the process. This confirms the correctness of the developed procedure for analyzing the interaction of injection and production wells, and also indirectly confirms the correctness of the implementation of the module for solving direct problems. Creating a hydrodynamic model, carrying out calculations, as well as analyzing mutual influence of wells, are implemented in an automated software-information system.

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Определение влияния нагнетательных скважин на добывающие в динамике их работы с использованием гидродинамического моделирования*

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Рассматривается способ для определения взаимовлияния группы добывающих и нагнетательных скважин с использованием модуля гидродинамического моделирования. Полученные результаты взаимовлияния широко используются в практике – от поддержания пластового давления до эффективной оптимизации заводнения. Для моделирования процесса фильтрации многофазной смеси используются метод конечных элементов и специальная технология балансировки потоков через границы ячеек конечноэлементной сетки. Метод балансировки основан на подборе корректирующих добавок, удовлетворя-

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ющих минимуму функционала баланса объемов фильтрующейся смеси с регуляризацией. В модели взаимовлияния скважин используются полученная в результате гидродинамического моделирования зависимость устьевого давления, а также объемы закачиваемой и/или отбираемой смеси (с разными знаками). Для интересующей добывающей скважины устьевое давление аппроксимируется функцией от объемов закачиваемой смеси на группе соседних скважин. Коэффициенты регрессии находятся с помощью метода наименьших квадратов. Приведены исследования применения метода на трех характерных задачах, эмулирующих работу добывающих и нагнетательных скважин. Для оценки степени влияния соседней скважины на интересующую скважину вводится критерий значимости (коэффициент влияния) в виде отношений остаточных сумм квадратов отклонений. Результаты расчетов свидетельствуют о том, что оцененное влияние скважин соответствует физике процесса – это подтверждает корректность работы разработанной процедуры анализа взаимовлияния нагнетательных и добывающих скважин, а также косвенно подтверждает правильность реализации модуля решения прямых задач. Задание гидродинамической модели, проведение расчетов, а также анализ взаимовлияния скважин реализованы в интерактивном режиме в автоматизированной программно-информационной системе.

Ключевые слова: гидродинамическое моделирование, фильтрация, метод конечных элементов, балансировка потоков, взаимодействие скважин, регрессия, влияние, информационная система

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