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Kinetic equations of creep and damage for description of materials with non-monotonic dependence of fracture strain on stress

Inna Banshchikova *

Lavrentyev Institute of Hydrodynamics of the Siberian Branch of the RAS, 15 Ac. Lavrentieva ave., Novosibirsk, 630090, Russian Federation

https://orcid.org/0000-0002-4045-7975, binna@ngs.ru

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ABSTRACT

Introduction. Reducing the level of damage accumulation during pressure treatment of materials at elevated temperatures in creep and close to superplasticity modes in the manufacture of parts can significantly increase its service life in the cold state. Finding temperature and power conditions leading to a reduction in damage of material during the production process and operation is an important task. The purposes of the work: 1) to show the possibility of using the Sosnin-Gorev creep and damage model for alloys with a non-monotonic dependence of strain at fracture on diagrams with creep curves; 2) to carry out comparative analysis of damage accumulation under conditions of uniaxial tension at constant stress and at constant strain rates for alloy with such a dependence. **Research methods.** Used scalar damage parameter is equated to the normalized deformation, i.e. to the ratio of the current strain to the fracture strain. To find the coefficients of relations creep and damage, the similarity of the creep curves in the normalized values "normalized strain – normalized time", i.e. the presence of single normalized curve of damage accumulation is checked. The least squares method is used to approximate the experimental data. Numerical integration methods are used for comparative analysis of deformation modes. **Results and discussion.** Determination of the parameters of the creep and damage equations by the method of a single normalized curve is carried out on the example of experimental data for steel 12Kh18N10T (12Cr18Ni10Ti) at 850 °C, which has a minimum of fracture strain in diagrams with creep curves. Analysis of the static and kinematic modes of deformation for studied material showed that damage accumulation in both cases is practically the same for stresses close to the stress at which this minimum is reached. If the stresses are lower, then the lower level of damage accumulation will be in the kinematic mode; if the stresses above the minimum value, then the static mode will lead to a lower level of damage accumulation. **Application.** The obtained results can be useful when choosing rational modes of forming structural elements from alloys with a non-monotonic dependence of the fracture strain on stress, as well as in evaluating it for long-term strength during operation.

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Introduction

To reduce the damage of materials during its pressure shaping, the modes of high-temperature creep and close to superplasticity have long been used. The choice of thermal-power loading modes that are rational from the point of view of damage accumulation in the manufacture of structural elements leads to an increase in the service life of the final product in the cold state. Finding such modes of shaping in the production of metal structures is an important task. Residual resource assessment during operation is another relevant area of research.

*** Corresponding author**
Banshchikova Inna A., D.Sc. (Physics and Mathematics), Senior researcher
Lavrentyev Institute of Hydrodynamics of Siberian Branch of RAS,
15, Ac. Lavrentieva ave.,
630090, Novosibirsk, Russian Federation
Tel.: 8 (383) 333-27-50, **e-mail:** binna@ngs.ru

Detailed reviews of creep models that take into account the accumulation of damage in the material are carried out in [1–4]. Models of damage accumulation are divided into phenomenological and physically based ones. The founder of the phenomenological approach is L. M. Kachanov [5]. He introduced the concepts of "continuity" or "cracking", describing the state of the material with one structural parameter $\psi(t)$ $(0 \le \psi \le 1$, t – time). The mechanisms of damage and the physical nature of this damage parameter are not specified. Later Yu. N. Rabotnov introduced the parameter $(0 \le q \le 1)$ "quite conditionally", assuming that when $q = 0$ the material is considered not damaged, and when $q = 1$ microscopic cracks begin to form, which actually means its destruction [6]. Even later Yu. N. Rabotnov generalizes the model by introducing several such damage parameters without giving it a specific physical meaning. Such parameters can describe various aspects of damage accumulation, for example, consider the aggressiveness of the environment [2].

Physically substantiated models take into account the microstructure of the material, the density of pores or dislocations in the process of damage accumulation [7–9]. Since most materials have anisotropic properties, the damage, as a rule, has a tensor or vector form [2, 4, 10]. However, until now, the introduction of vectors and damage tensors into the models is limited, since the calculations are significantly complicated. Various creep models with a scalar damage parameter are actively used to this day, and the introduction of the corresponding equivalent stress into the equations in some cases makes it possible to take into account the presence of anisotropy properties. According to the model of Yu. N. Rabotnov, the constitutive relations for the uniaxial stress state have the following form [11]:

$$\frac{d\varepsilon^c}{dt} = f_c(\sigma, T, q_1, q_2, ..., q_n), \qquad \frac{dq_i}{dt} = \varphi_c(\sigma, T, \varepsilon^c, t, q_1, q_2, ..., q_n), \tag{1}$$

where ε^c – irreversible creep strains, T – temperature, t – time, q_i – structural parameters. In the case of a single damage parameter q, the system (1) can be concretized in the following form [6]:

$$\frac{d\varepsilon^c}{dt} = \frac{B_\varepsilon \sigma^n}{(1-q)^{\kappa_1}}, \qquad \frac{dq}{dt} = \frac{B_\omega \sigma^g}{(1-q)^{\kappa_2}}.$$

Here the parameters B_ε, B_ω, n, g, κ_1, κ_2 are determined on the basis of experimental data and generally depend on the temperature. It should be noted that this system of equations has arbitrariness, since it is impossible to determine the parameters of the equations from experimental data independently of each other [6]. There is no uniform method for determining the parameters. There is no unified method for determining the parameters, and when choosing it, researchers, as a rule, are guided by the desire to describe the experimental data as best as possible.

To describe creep and damage accumulation, authors [12] introduce the value of the dissipation power $W_A = \dot{\varepsilon}^c_{ij}\sigma_{ij}$, where ε^c_{ij}, σ_{ij} are the components of the creep strain and stress tensors (symbol "point" denotes the derivative with respect to time t), while it is assumed that the work of dissipation at the time of fracture is constant $A_* = \text{const}$ (energy approach in the version of O. V. Sosnin). Using the phenomenological approach of Yu. N. Rabotnov to describe deformation, authors [13] generalize the application of the energy approach of the kinetic equations on the case in which the creep strain at fracture time isn't constant value $\varepsilon^c_* \ne \text{const}$ $(A_* \ne \text{const})$.

This paper demonstrates the possibility of describing deformation processes using the Sosnin - Gorev model [13] in the case when the function $\varepsilon^c_*(\sigma)$ on the "strain – time" creep diagrams at stresses $\sigma = \text{const}$ is non-monotonic. The method of determining the parameters of the constitutive equations of creep and damage is described.

The choice of deformation modes in order to reduce the level of damage accumulation to increase the product life during production and operation is an urgent task. The papers of I. Y. Tsvelodub and

K. S. Bormotin theoretically and numerically substantiate the use of kinematic modes with a constant creep strain rate within the framework of the energy approach $A_* = \text{const}$ [14–16]. However, structural alloys can be described by an energy variant of the creep theory $A_* = \text{const}$ in a fairly narrow range of rates and temperatures.

In [17], in order to assess the residual operational life, two deformation modes under uniaxial tension conditions were studied for alloys having a monotonic dependence of the ultimate strain (fracture strain) on stress (AK4-1 (Al–Cu–Mg–Fe–Ni), 250°C; D16T (Al–Mg–Cu), 250°C; VT9 (Ti–Al–Mo–Zr), 600°C; steel 09G2S-12 (Fe–Si–Cu–Cr–Ni–C), 730°C; 3V (Ti–Al–V), 20°C). The deformation modes at constant stresses and at constant strain rates corresponding to these stresses were compared. It is shown analytically and numerically that if the dependence on creep diagrams $\varepsilon_*^c(\sigma)$ decreases monotonically with increasing σ, then the accumulation of damage is less in kinematic modes $\eta = d\varepsilon^c / dt = B_\varepsilon \sigma^{\text{n}} = \text{const}$. Such materials include alloys that are described by the energy approach of the creep theory $A_* = \text{const}$ and the condition $g \geq n$ is satisfied. For alloys, in which dependence $\varepsilon_*^c(\sigma)$ monotonically increases in diagrams with creep curves $\varepsilon^c(t)$, the accumulation of damage is less in the mode $\sigma = \text{const}$.

The purpose of this work is to carry out a comparative analysis of two deformation modes of tensile rods for an alloy with a non-monotonic dependence of the ultimate strain using the kinetic equations of creep and damage: static $\sigma = \text{const}$ and kinematic $\eta = B_\varepsilon \sigma^{\text{n}} = \text{const}$.

Theory and methods

Constitutive relations of creep and damage

Equations (1) in [13] are defined as

$$\frac{dA}{dt} = \frac{f_A(\sigma, T)}{(1-q)^{\kappa_1}}, \qquad \frac{dq}{dt} = \frac{\Phi_c(\sigma, T)}{(1-q)^{\kappa_2}}, \qquad (0 \leq q \leq 1). \tag{2}$$

Here $A = \int_0^t W_A dt = \int_0^{\varepsilon^c} \sigma_{ij} d\varepsilon_{ij}^c$.

Replacing q by $\left(1-(1-\omega)^{1/(\kappa_2-\kappa_1+1)}\right)$, the relations (2) can be reduced to the following [13]

$$\frac{dA}{dt} = \frac{f_A(\sigma, T)}{(1-\omega)^m}, \qquad \frac{d\omega}{dt} = \frac{\varphi_c(\sigma, T)}{(1-\omega)^m}, \tag{3}$$

thus eliminating the arbitrariness in determining the coefficients of the constitutive relations. Under conditions of a uniaxial state, the parameter ω $(0 \leq \omega \leq 1)$ must satisfy the equation of a single normalized curve

$$(1-\omega)^{m+1} = (1-\tilde{\tau}), \tag{4}$$

where $\tilde{\tau} = (\kappa_2+1)\int_0^t \Phi_c(\sigma, T)dt$ or $\tilde{\tau} = (m+1)\int_0^t \varphi_c(\sigma, T)dt$ is the normalized time.

Integrating (4), for we obtain

$$\omega = 1 - (1-(m+1)\int_o^t \varphi_c(\sigma, T)dt)^{\frac{1}{m+1}}. \tag{5}$$

For $\sigma = \mathrm{const}$ from (5) we have

$$\omega = 1 - [1 - (m+1)\ \varphi_c(\sigma, T)\ t]^{\frac{1}{m+1}}, \qquad A = \frac{f_A(\sigma, T)}{\varphi_c(\sigma, T)}\omega, \tag{6}$$

$$\omega = A / A_*, \qquad A_* = f_A(\sigma, T) / \varphi_c(\sigma, T), \tag{7}$$

$$\tilde{\tau} = t / t_*, \qquad t_* = \frac{1}{(m+1)\ \varphi_c\ (\sigma, \mathrm{T})}. \tag{8}$$

Here t_* is the time of fracture.

If stress $\sigma = \mathrm{const}$, then damage parameter is $\omega = A / A_* = \varepsilon^c / \varepsilon_*^c$ and the verification of equations (3) should be carried out in the normalized values $\omega = A / A_* = \varepsilon^c / \varepsilon_*^c$, $\tilde{\tau} = \mathrm{t/t}_*$.

If the material, in addition to the stages of steady-state creep and softening, has a hardening stage, then (3) can be rewritten as [13]

$$W_A = \frac{dA}{dt} = \frac{f_A(\sigma, T)}{\omega^\alpha (1 - \omega^{\alpha+1})^m}, \qquad \frac{d\omega}{dt} = \frac{\varphi_c(\sigma, T)}{\omega^\alpha (1 - \omega^{\alpha+1})^m}, \tag{9}$$

where α is the hardening parameter. In this case, in normalized values $\omega = A/A_* \equiv \varepsilon^c/\varepsilon_*^c$, $\tilde{\tau} = t/t_*$, equation of a single normalized curve must also be fulfilled for ω in the form

$$(1 - \omega^{(\alpha+1)})^{m+1} = 1 - \tilde{\tau}. \tag{10}$$

Integrating (9) with $\sigma = \mathrm{const}$ instead of (6) we obtain

$$\omega = \left(1 - \left(1 - (m+1)(\alpha+1)\varphi_c(\sigma, T)t\right)^{\frac{1}{m+1}}\right)^{\frac{1}{1+\alpha}}, \qquad A = \frac{f_A(\sigma, T)}{\varphi_c(\sigma, T)}\omega,$$

$$\varepsilon^c = \frac{f_A(\sigma, T)}{\sigma \cdot \varphi_c(\sigma, T)}\omega, \qquad \varepsilon_*^c = \frac{f_A(\sigma, T)}{\sigma \cdot \varphi_c(\sigma, T)}. \tag{11}$$

It follows from the analysis of (11) that in the uniaxial case the parameter characterizes the deformability of the material, i.e. $\omega = \varepsilon^c/\varepsilon_*^c$ – reduced deformation, and the dependence ε_*^c on stress σ can be arbitrary.

In the case of a complex stress state, equations (9) can be generalized [13, 18]:

$$W_A = \frac{dA}{dt} = \frac{f_A(\sigma_e, T)}{\omega^\alpha (1 - \omega^{\alpha+1})^m}, \qquad A = \int_{\mathrm{o}}^{\mathrm{t}} \sigma_{\mathrm{ij}} \dot{\varepsilon}_{ij}^c dt, \tag{12}$$

$$\frac{d\omega}{dt} = \frac{\varphi_c(\sigma_{e*}, T)}{\omega^\alpha (1 - \omega^{\alpha+1})^m}, \qquad 0 \le \omega \le 1, \tag{13}$$

$$\eta_{ij} = \frac{d\varepsilon_{ij}^c}{dt} = \lambda \frac{\partial \sigma_e}{\partial \sigma_{ij}}, \qquad \lambda = \frac{W_A}{\sigma_e}. \tag{14}$$

Here σ_e, σ_{e*} are the equivalent stresses.

The stress σ_e .can be taken, for example, the stress intensity according to Mises $\sigma_e = \sigma_i = (3\bar{\sigma}_{ij}\bar{\sigma}_{ij} / 2)^{1/2}$, $\bar{\sigma}_{ij}$ – the components of the stress deviator. The choice of an equivalent stress (the criterion of long-term strength), as already noted, allows us to take into account the anisotropic nature of damage accumulation for various stress states. The analysis of the criteria for long-term creep strength is given in [19–21].

Entering normalized values $\omega = A / A_* = \varepsilon^c / \varepsilon^c_*$, $\tilde{\tau} = t/t_*$ in the analysis of experimental creep curves makes it possible to determine the parameter ω through the values ε^c, t, ε^c_*, t_* measured in the experiment, while it remains in no way related to the microstructure of the material (with changes in the density of pores, dislocations, vacancies). The fracture in the experiment is understood as the separation of the sample into parts.

The geometric similarity of the curves (10) at constant stresses in the normalized values was obtained for a number of alloys [12, 13, 22–24]. Publications [25, 26] demonstrate the possibility of using equations (12)–(14) to describe materials with a monotonic dependence $\varepsilon^c_*(\sigma)$ in creep diagrams $\varepsilon^c(t)$. In [25] this is shown by the example of torsion of rods made of an alloy without the first creep stage ($\alpha = 0$) AK4-1 (Al-Cu-Mg-Fe-Ni) at T = 250 °C, while the value of the ultimate strain intensity $\varepsilon^c_{i*}(\sigma_i)$ increases monotonically. Publication [26] studied a titanium alloy 3B (Ti–Al–V) at T = 20 °C, which has all three pronounced creep stages. Equations (12)–(14) in the variant $A_* = \text{const}$ describe it, while the ultimate strain intensity, on the contrary, monotonically decreases with increasing of σ_i. The experimental data of both alloys are densely located near the "single curve". The possibility of such grouping into a normalized curve of test data with a non-monotonic dependence $\varepsilon^c_{i*}(\sigma_i)$ is discussed in [22, 23], but the procedure for obtaining the parameters of equations (12)–(14) is not given.

Method for determining the parameters of kinetic equations

Publications [12, 24, 27] discuss the methods for finding the coefficients of kinetic equations (12)–(14). As a rule, the investigated alloys have a monotonic dependence $\varepsilon^c_*(\sigma)$ on the experimental diagrams. If the dependence is non-monotonic, then in these works it is usually averaged and assumed to be monotonic.

The exponent *m* describes softening and is determined by the third section of a single normalized creep curve: if the material has a hardening stage, then after the inflection point located on steady-state section; if the first stage is absent, then only along the last section. If $\alpha = 0$, then after taking the logarithm (4) we have

$$(m+1)\ln(1-\omega) = \ln(1-\tilde{\tau}).$$

This relation is the equation of a straight line in logarithmic coordinates $\ln(1-\omega) - \ln(1-\tilde{\tau})$. Its slope determines the exponent *m*. Here $\omega = \varepsilon^c_k / \varepsilon^c_{k*}$; $\tilde{\tau} = t_k / t_{k*}$; index *k* means the number of the creep curve $\sigma = \sigma_k = \text{const}$; ε^c_k, t_k are the creep strain and time at the point of transition to the third stage; ε^c_{k*}, t_{k*} are the ultimate strain and fracture time of the sample. The parameter *m* is found by averaging its values obtained at different values $\sigma = \sigma_k$. If $\alpha = 0$, then the parameter *m* in (4) can be found by the least squares method applied to experimental points in normalized coordinates.

Publication [27] demonstrates the obtaining of the exponent α on the basis of experimental data on tension and compression using the dependence of the type of the strain hardening ($\dot{\varepsilon}^c = \varepsilon^{-\alpha} f_c(\sigma, T)$). When found α in (9), it is accepted $\frac{d\omega}{dt} = \varphi_c(\sigma, T) / \omega^\alpha$. Integrating this equation from $\omega = 0$ to the current values ω и *t*, after subsequent logarithm, we come to the equation of the straight line

$$(\alpha+1)\ln(\omega) = \ln[(\alpha+1)\varphi_c(\sigma, T)] + \ln(1-\tilde{\tau}).$$

The slope of this straight line similarly to the exponent *m* determines the exponent α: after processing and averaging the experimental data of the first sections of the creep curves at $\sigma = \sigma_k = \text{const}$ or according to the data of the normalized curve $\omega(\tilde{\tau})$ to the transition point to the steady-state stage by the least squares method.

Dependences of a power-law or exponential form can be selected as functions $f_c(\sigma, T)$ and $\varphi_c(\sigma, T)$ [2, 13]: $B\sigma^n$; $B_1 \exp(\beta\sigma)$; $B_3(\exp(\beta\sigma^2) - 1)$ and etc. If $f_c(\sigma, T) = B_\varepsilon \sigma^n$, then the coefficients B_ε, n are found from the experimental data at the steady-state stage of the curve $\varepsilon^c(t)$ at $\sigma = \text{const}$ (in this case, in (12)–(14) $f_A(\sigma, T) = B_A \sigma^{n+1}$, $B_A = B_\varepsilon$). After taking the logarithm of the ratio $\dot{\varepsilon}^c = B_A \sigma^n$, we get $\ln(\dot{\varepsilon}^c) = \ln(B_A) + n \ln \sigma$. Averaging the *n* obtained for different $\sigma = \sigma_k = \text{const}$, we calculate the values of the coefficients *n* and B_A. If $\varphi_c(\sigma, T) = B_\omega \sigma^g$, then from (11) it is follows $t_* = 1 / \left((m+1)(\alpha+1) B_\omega \sigma^g\right)$. Taking the logarithm of the last expression, we obtain the equation of the straight line $\ln(t_*) = -\ln((m+1)(\alpha+1)B_\omega) - g\ln(\sigma)$ for finding the coefficients g, B_ω.

The functions $f_c(\sigma, T)$ and $\varphi_c(\sigma, T)$ taken in a power-law form allow us to describe the deformation of materials *with a monotone dependence of the ultimate dissipation work* A_* *(strain* ε_*^c) on stress. Publications [25, 26] shown this using the example of alloys AK4-1 (Al–Cu–Mg–Fe–Ni) at $T = 250$ °C and 3V (Ti–Al–V) at $T = 20$ °C, which were satisfactorily described by power functions within the approach of a single normalized damage curve.

Analysis of the creep tests results shows that the function $\varepsilon_*^c(\sigma)$ for some alloys may be non-monotonic, namely, in a certain stress range have a minimum (12Cr18Ni10Ti, 850°C; 15Cr1Mo1V, 565 °C) or a maximum (Ti-Al-Mn, 500°C; Al-Zn-Mg-Cu, 165ºC; Al-Mg-Mn, 165ºC; Al-Mg-Sc, 500 °C) [22, 23, 28–31]. The energy model of the kinetic equations of creep and damage in the initial version $A_* = \text{const}$ is applicable only in a narrow range of temperatures and stresses. The non-monotonic form of the function $\varepsilon_*^c(\sigma)$ also complicates the description of deformation processes using models that take into account the microstructure. Authors of [2, 22, 28, 30] discuss the possibility of a mathematical description of such materials using a phenomenological approach.

Let consider a technique for finding the coefficients of kinetic creep equations with a scalar damage parameter for materials with a non-monotonic function $\varepsilon_*^c(\sigma)$. Publication [28] studied steel 12Cr18Ni10Ti at $T = 850$ °C. Uniaxial tensile tests were carried out on the equipment of the Institute of Mechanics of Moscow State University. The monograph by A. M. Lokoshchenko [32] gives a description of a typical IMekh-5 device used for carrying out experiments on tension and torsion at creep. The experiments were carried out on tubular samples with an outer diameter of 12 mm, a wall thickness of 0.5 mm and a working length of 70–100 mm at a constant temperature of 850 ° C. In the tests, a constant acting load was applied to the sample. The change in the cross-section during creep was assumed to be insignificant, and therefore it was assumed that at a constant load, the stress in the cross-section is constant until fracture. The original curves at $\sigma = 39{,}2;\ 49;\ 58{,}8;\ 78{,}4$ MPa are published in [33]. From 2 to 8 tests were performed for each stress. The averaged results [28] showed that at a stress of $\sigma \approx 60$ MPa, the dependence $\varepsilon_*^c(\sigma)$ has a minimum in diagrams with creep curves $\varepsilon^c(t)$. Similarly to [28], we will use the functions $f_A(\sigma) = B_A \sigma \cdot \mathrm{sh}(\sigma / c)$ and $\varphi_c(\sigma) = B_\omega \sigma^g$ to determine the coefficients in equations (12)–(14). Note that the authors [28] used a dependence of type (2) with different exponents in the denominators to approximate the experimental data. We assume that there is no hardening, i.e. $\alpha = 0$. It follows from (11) that the fracture strain is

$$\varepsilon_*^c(\sigma) = \frac{B_A}{B_\omega} \cdot \frac{\mathrm{sh}(\sigma / c)}{\sigma^g}. \tag{15}$$

Differentiating twice $\varepsilon_*^c(\sigma)$, we get

$$\frac{d^2\varepsilon_*^c}{d\sigma^2} = \frac{B_A}{B_\omega \sigma^{g+1}} \left[\left(\frac{\sigma}{c^2} + \frac{g(g+1)}{\sigma} \right) \mathrm{sh}\left(\frac{\sigma}{c}\right) - \frac{2g}{c} \mathrm{ch}\left(\frac{\sigma}{c}\right) \right]. \tag{16}$$

Let transform the expression in the square brackets of the right side of (16). We decompose the hyperbolic sine and cosine into a Taylor series and group the coefficients at the same degrees, omitting the multiplier before the bracket:

$$1 \ : g(g-1)/c;$$

$$\sigma^2 : \frac{1}{c^3}\left(1 + \frac{g(g+1)}{3!} - \frac{2g}{2!}\right)$$

$$\sigma^4 : \frac{1}{c^5}\left(\frac{1}{3!} + \frac{g(g+1)}{5!} - \frac{2g}{4!}\right)$$

....

$$\sigma^{2k} : \frac{1}{c^{2k+1}}\left(\frac{1}{(2k-1)!} + \frac{g(g+1)}{(2k+1)!} - \frac{2g}{(2k)!}\right) \qquad (k \in \mathrm{N})$$

The obtained coefficients at σ^{2k} can be generalized in a following form

$$a_{2k} = \frac{1}{c^{2k+1}(2k+1)!}\left[(g-2k)^2 - g + 2k\right], \qquad k = 0, 1, \ldots, N. \tag{17}$$

As a rule, the coefficient $g > 2$. The coefficients a_{2k} are always greater than zero, except for the case when the parameter g is in the range $2k < g < 2k+1$. In other words, all coefficients a_{2k} are positive, except for one. However, the contribution of this negative term to the total sum in the required range of stresses and values of parameter c is small compared to the rest of the terms, and we can assume that $\frac{d^2\varepsilon_*^c}{d\sigma^2} \geq 0$. Note that if g turns out to be in the range $2k-1 < g < 2k$, then the expression in square brackets (17) is always greater than zero and all the terms of the series are positive. From the condition $\frac{d\varepsilon_*^c}{d\sigma} = 0$ we get the equation $\sigma \cdot \mathrm{cth}(\sigma / c) = g \cdot c$, solving which we find the minimum.

Taking the logarithm of the expression for the creep strain rate at the steady-state stage $\eta = B_A \mathrm{sh}(\sigma / c)$, we obtain the equation of the straight line for finding the parameters B_A and c: $\ln(\eta) \approx \sigma / c + \ln(B_A/2)$.

From (8) (or (11) at $\alpha = 0$) follows

$$t_* = 1 / \left((m+1) B_\omega \sigma^g\right). \tag{18}$$

After taking the logarithm (18), the equation of the straight line for finding g, B_ω is rewritten as: $\ln(t_*) = -\ln((m+1)B_\omega) - g\ln(\sigma)$.

Modes of deformation under tension of rods

Let's consider the process of damage accumulation for two modes of deformation under tension of rods made of an alloy with a non-monotonic dependence $\varepsilon_*^c(\sigma)$ on the example of steel 12Cr18Ni10Ti. Elastic strains are neglected. In view of the fact that in (12)–(14) $f_A(\sigma) = B_A\sigma \cdot \mathrm{sh}(\sigma / c)$ and $\varphi_c(\sigma) = B_\omega\sigma^g$, the expression for the creep strain rate is written as:

$$\frac{d\varepsilon^c}{dt} = \frac{B_A \mathrm{sh}(\sigma / c)}{\omega^\alpha(1-\omega^{\alpha+1})^m}. \tag{19}$$

In the case of *mode 1* ($\sigma_i = \sigma_0 = \mathrm{const}$), it follows from (11) that

$$\omega = \left(1 - \left(1 - (m+1)(\alpha+1)B_\omega\sigma_0^g t\right)^{1/(m+1)}\right)^{1/(1+\alpha)}$$

and

$$\varepsilon^c(t) = \frac{B_A}{B_\omega}\frac{\mathrm{sh}(\sigma_0 / c)}{\sigma_0^g}\omega(t). \tag{20}$$

In the case of *mode 2* ($\eta_0 = B_A \mathrm{sh}(\sigma_0 / c) = \mathrm{const}$), it follows from (12), (14) that

$$\mathrm{sh}(\sigma / c) = \left(1-\omega^{\alpha+1}\right)^m \omega^\alpha \eta_0 / B_A$$

and

$$\sigma = c\ln\left(F_n + \sqrt{F_n^2 + 1}\right), \tag{21}$$

where $F_n(\omega) = \left(1-\omega^{\alpha+1}\right)^m \omega^\alpha \eta_0 / B_A = \left(1-\omega^{\alpha+1}\right)^m \omega^\alpha \mathrm{sh}(\sigma_0 / c)$. Substituting the expression for σ in (13), we obtain the equation for finding of ω:

$$\frac{d\omega}{dt} = \frac{B_\omega}{\omega^\alpha(1-\omega^{\alpha+1})^m}\left(c\ln\left(F_n(\omega) + \sqrt{(F_n(\omega))^2 + 1}\right)\right)^g. \tag{22}$$

To numerically solve (21), (22), one can use, for example, the Runge-Kutta method.

The number of deformation modes can be considered much more. In [34], in relation to the problems of shaping a hemispherical shell from a flat workpiece, the modes of deformation under the action of constant pressure, linearly increasing pressure, or when the law of variation of the deflection in time is specified is investigated.

Results and discussion

Determination of the parameters of steel 12Cr18Ni10Ti

Experimental data for steel 12Cr18Ni10Ti show that the fracture strain ε_*^c in the stresses range from 40 MPa to 80 MPa first decreases monotonically with increasing stress, and at $\sigma_0 \approx 60$ MPa begins to increase monotonically. In Fig. 1, *a* points 1–4 show the experimental dependences $\varepsilon^c(t)$ corresponding to

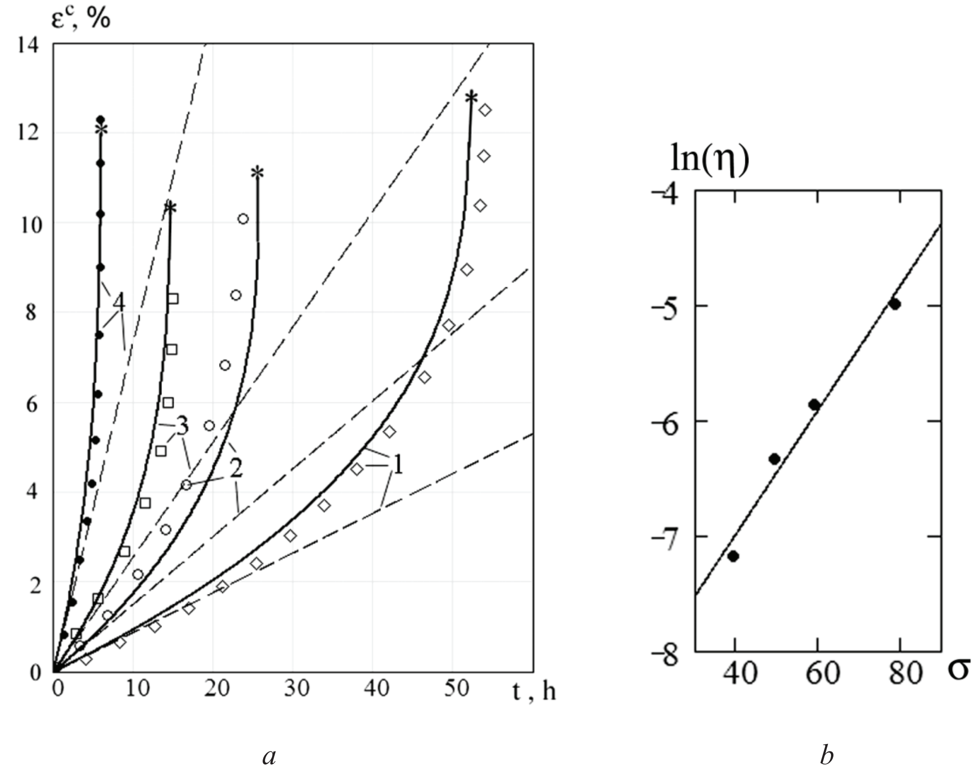

Fig. 1. Experimental data (points) and approximation dependences (lines) 1–4 for steel 12Cr18Ni10Ti at temperature T = 850 °C and σ = 39.2; 49; 58.8; 78.4 MPa. Dashed lines – approximation at steady-state creep; solid lines – approximation with considering the damage (*a*); experimental data (points) and approximation (line) in the coordinates "*ln* (η) – σ" corresponding to the steady-state creep (*b*)

$\sigma = 39{,}2;\ 49;\ 58{,}8;\ 78{,}4$ MPa. In Fig. 1, *b* these data are rebuilt in the coordinates $\ln(\eta) - \sigma$ to find the coefficients of the second creep stage. It can be seen that the experimental points are located near a straight line $\ln(\eta) = a\sigma + b$. The coefficients a and b were found using the least squares method. Then the coefficients $c = 1/a$ and $B_A = 2\exp(b)$ were determined. As a result, the values of $B_A = 2{,}183 \cdot 10^{-4}$ h^{-1} and $c = 18{,}6$ MPa were obtained; the dashed lines in Fig. 1,*a* is an approximation by the dependence $\eta = B_A \mathrm{sh}(\sigma / c)$ with the found values B_A and c. Pearson's correlation coefficient (linear pair correlation coefficient) is $k_p = 0{,}987$.

In Fig. 2, *a*, the experimental data are rebuilt in the normalized coordinates $\omega - \tilde{\tau}$ ($\tilde{\tau} = t / t_*$, $\omega = \varepsilon / \varepsilon_*^c$, $0 \le \tilde{\tau} \le 1$). The solid line is an approximation of this data by a "single curve" (4) using the least squares method. Coefficient m = 1.8 was obtained with the correlation index of nonlinear regression k_r = 0.979.

The straight line in Fig. 2, *b* is an approximation of the experimental data of steel 12Cr18Ni10Ti, obtained by the method of least squares in coordinates $\ln(t_*) - \ln(\sigma)$. The coefficients g = 3.165 and B_ω = 6.231·10^{-8} MPa^{-g}h^{-1} were determined from (18). Pearson's correlation coefficient is $k_p = 0{,}998$. Thus, all the parameters of equations (12)–(14) have been found.

Solid lines 1–4 in Fig. 1, *a* are an approximation of experiments using equations (12)–(14), where $f_c(\sigma) = B_A \sigma \cdot \mathrm{sh}(\sigma / c)$ and $\varphi_c(\sigma) = B_\omega \sigma^g$ in view of coefficients found. The values ε_*^c = 12.9, 10.9, 10.4, 12.0 % are fracture strains calculated according to (15) at σ = 39.2; 49; 58.8; 78.4 MPa.

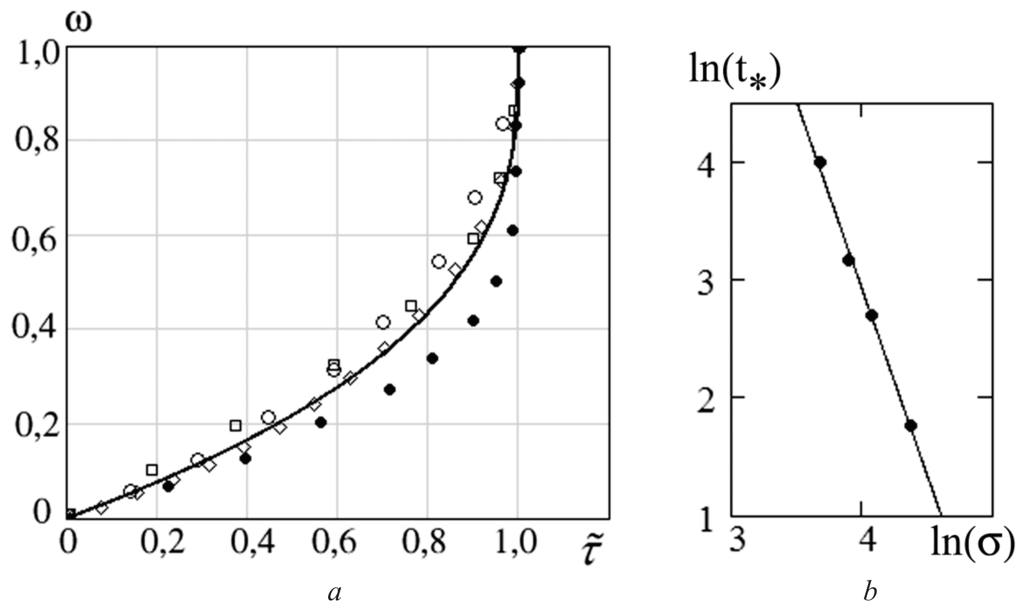

Fig. 2. Normalized accumulation damage curve for steel 12Cr18Ni10Ti: experimental data (points) and its approximation (line) (*a*); experimental data (points) and approximation (line) in logarithmic coordinates $\ln(t_*) - \ln(\sigma)$ (*b*)

Comparative analysis of deformation modes

Let's consider the uniaxial stretching of a rod made of steel 12Cr18Ni10Ti to a given strain value ε_0^c for two deformation modes.

For *mode 1* from (20) we have

$$\omega(\sigma_0) = \omega_\sigma\left(\varepsilon_0^c, \sigma_0\right) = \frac{B_\omega}{B_A}\frac{\sigma_0^g}{\mathrm{sh}(\sigma_0 / c)}\varepsilon_0^c. \tag{23}$$

For *mode* 2 in view of the time $t = \dfrac{\varepsilon_0^c}{\eta_0} = \dfrac{\varepsilon_0^c}{B_A \mathrm{sh}(\sigma_0 / c)}$ and solving (21), (22) we find numerically

$$\omega(\sigma_0) = \omega_\eta\left(\varepsilon_0^c, \sigma_0\right). \tag{24}$$

The creep rates η_0 = $8.85 \cdot 10^{-4}$; $1.5 \cdot 10^{-3}$; 2.57·10-3; $7.39 \cdot 10^{-3}$ h^{-1} corresponds to the stresses σ_0 = 39.2; 49; 58.8; 78.4 MPa at the steady-state stage. Lines 1–4 in Fig. 3, *a* are dependences $\sigma(t)$ obtained from the solution of the system (21), (22) for these four kinematic deformation modes $\eta_0 = \mathrm{const}$. It can be seen that the stage of steady-state creep in Fig. 1,*a* is very short, as a result, the curves in Fig. 3,*a* haven't horizontal part and immediately begin to fall. Up to the fracture in mode 2, it is actually impossible to perform calculations. This can be explained by the fact that at low stress values, the fracture strain begins to increase significantly. For example, according to (15) at $\sigma_0 = 20$ MPa the strain is $\varepsilon_*^c(\sigma_0) = 35\,\%$, and at $\sigma_0 = 15$ MPa the fracture strain is already 60 %. The mode close to relaxation mode begins to be observed (Fig. 3, *a*) at such low values of stresses. However, there are no experimental data at such stresses, so it can be assumed that the model adequately describes the deformation in the stress range of 40 MPa $\le \sigma \le 80$ MPa. For a more accurate description of the deformation in a wider stress range, it may be necessary to enter a second scalar parameter or additional coefficients.

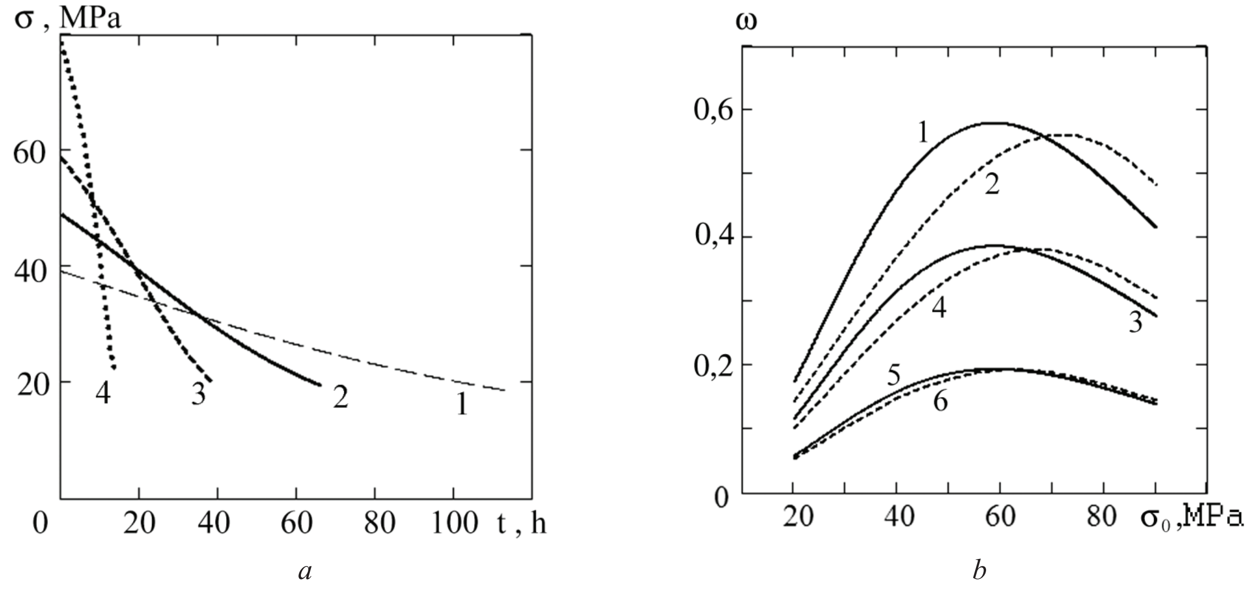

Fig. 3. Steel 12Cr18Ni10Ti, 850 ºC. Lines 1–4 dependences σ(*t*) for the kinematic mode $\eta_0 = B_A \mathrm{sh}(\sigma_0 / c) =$ = const for σ_0 = 39.2; 49; 58.8; 78.4 MPa (*a*);dependences $\omega(\sigma_0)$: solid lines 1, 3 – static deformation mode; dashed lines 2, 4 – kinematic deformation mode; lines 1, 2 correspond ε_0 = 6 %, , lines 3, 4 – ε_0 = 4 %, lines 5, 6 – ε_0 = 2 % (*б*)

Solid lines 1, 3, 5 in Fig. 3, *b* are numerical calculation of $\omega(\sigma_0)$ according to the formula (23); dashed lines 2, 4, 6 are calculation of $\omega(\sigma_0)$ according to the formula (24); lines 1, 2 correspond to the strain $\varepsilon_0^c = 6\,\%$, lines 3, 4 correspond to the strain $\varepsilon_0^c = 4\,\%$, lines 5, 6 correspond to the strain $\varepsilon_0^c = 2\,\%$. It can be seen from the analysis of the graphs that for both modes the accumulation of damage at $\sigma_0 \approx 65$ MPa is almost the same; mode 2 is preferable at $\sigma_0 < 65$ MPa, since $\omega_\eta < \omega_\sigma$; and mode 1 is the best at $\sigma_0 > 65$ MPa, since $\omega_\sigma < \omega_\eta$.

It can be assumed that for alloys with a maximum of function $\varepsilon_*^c(\sigma)$ in diagrams with creep curves $\varepsilon^c(t)$ [23, 29–31], deformation modes with rates corresponding to stresses from the interval at which this maximum is reached, on the contrary, will give the least accumulation of damage, while strains will be maximum. In fact, such modes can be classified as modes close to superplasticity.

Conclusions

1. The research showed the possibility of using creep equations with a scalar damage parameter in Sosnin-Gorev approach for alloys with a non-monotonic dependence of fracture strain on stress in diagrams with creep curves. The damage parameter is equated to the normalized strain, namely, to the ratio of the current strain to the fracture strain.

2. To determine the coefficients of the kinetic creep equations, it is necessary to check the geometric similarity of the experimental creep curves in the normalized values “normalized strain – normalized time”, i.e. the presence of a single normalized damage accumulation curve. The determination of the coefficients using the “single curve” method is demonstrated by the example of experimental data for steel 12Cr18Ni10Ti at 850°C, which has a minimum of fracture strain in diagrams with creep curves at constant stress.

3. For steel 12Cr18Ni10Ti, which has a minimum of fracture strain on creep diagrams, the damage parameter was calculated for two modes of uniaxial deformation: when the stress in the cross section is constant and when the strain rate corresponding to these stresses at the steady-state creep stage is constant. The analysis of the deformation modes for the material under study showed that the accumulation of damage

in both cases is practically the same for stresses at which this minimum is reached. If the stress is less than the minimum value, then the accumulation of damage is less in the kinematic deformation mode; if the stress is greater, the accumulation of damage is less in the static mode. The obtained conclusions about the advantages of deformation methods should be taken into account when choosing the modes of shaping structures made of alloys with a non-monotonic dependence of the fracture strain on stress, as well as when evaluating it for long-term strength during operation.

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Conflicts of Interest

The author declare no conflict of interest.