Obrabotka metallov (tekhnologiya, oborudovanie, instrumenty) = Metal Working and Material Science. 2022 vol. 24 no. 4 pp. 98–112
ISSN: 1994-6309 (print) / 2541-819X (online)
DOI: 10.17212/1994-6309-2022-24.4-98-112

NSTU NETI

Obrabotka metallov -
Metal Working and Material Science

Journal homepage: http://journals.nstu.ru/obrabotka_metallov

Application of the synergistic concept in determining the CNC program for turning

Vilor Zakovorotny [a], *Valery Gvindjiliya* [b, *], *Ellina Fesenko* [c]

Don State Technical University, 1 Gagarin square, Rostov-on-Don, 344000, Russian Federation

[a] https://orcid.org/0000-0003-2187-9897, vzakovorotny@dstu.edu.ru, [b] https://orcid.org/0000-0003-1066-4604, sinedden@yandex.ru,
[c] https://orcid.org/0000-0003-0833-2758, ellinochaa@gmail.com

ARTICLE INFO

Article history:
Received: 08 September 2022
Revised: 03 October 2022
Accepted: 28 October 2022
Available online: 15 December 2022

Keywords:
Controlled cutting system
Synergetics of machining
Dynamic of the cutting process.

Acknowledgements
Research were partially conducted at core facility "Structure, mechanical and physical properties of materials".

ABSTRACT

Introduction. One of the dynamically developing areas of increasing the efficiency of *CNC* machines is associated with the use of the synergetic concept in determining the *CNC* program. The principle of compression-expansion of the dimensionality of the state space is used. **Subject.** On the example of the workpiece machining, the stiffness parameters of which are a function of the toolpath, all stages of control synthesis, which ensures the mutual consistency of dynamic subsystems, including the cutting process, are described in the paper. **The aim of the work is** to determine asymptotically stable machine actuator toolpath, given by *CNC* program parameters, from the set of paths, for which the condition of minimum wear intensity is fulfilled. **Method and methodology.** Mathematical modeling of the controlled cutting system, which is based on the principle of compression-expansion state space, is presented. When the dimension of the state space is expanded, the model of the dynamic cutting system includes all elements from the *CNC* system that programs the motion of the actuating elements to the elastic deformations of the tool, which interacts with the workpiece through the connection formed by the cutting process. The dynamic coupling integrates the subsystems into a single coupled control system. In this space, the desired shaping motion path of the tool tip relative to the workpiece is constructed, which should be the attractor of the entire state space. The transformation of the desired motion path into an attractor characterizes the procedure of compressing the dimensionality of the state space. It is supposed that it is possible to control the motion trajectories of the actuators within the bandwidths of the servomotors. **Results and Discussion.** The analysis of the stability of the cutting process is performed; the example of the efficiency of a *NC* program on the basis of the synergetic paradigm is presented. It is shown that by coordinating the external control with the internal dynamics of the system it is possible to increase the efficiency of a part production up to two times in machine time.

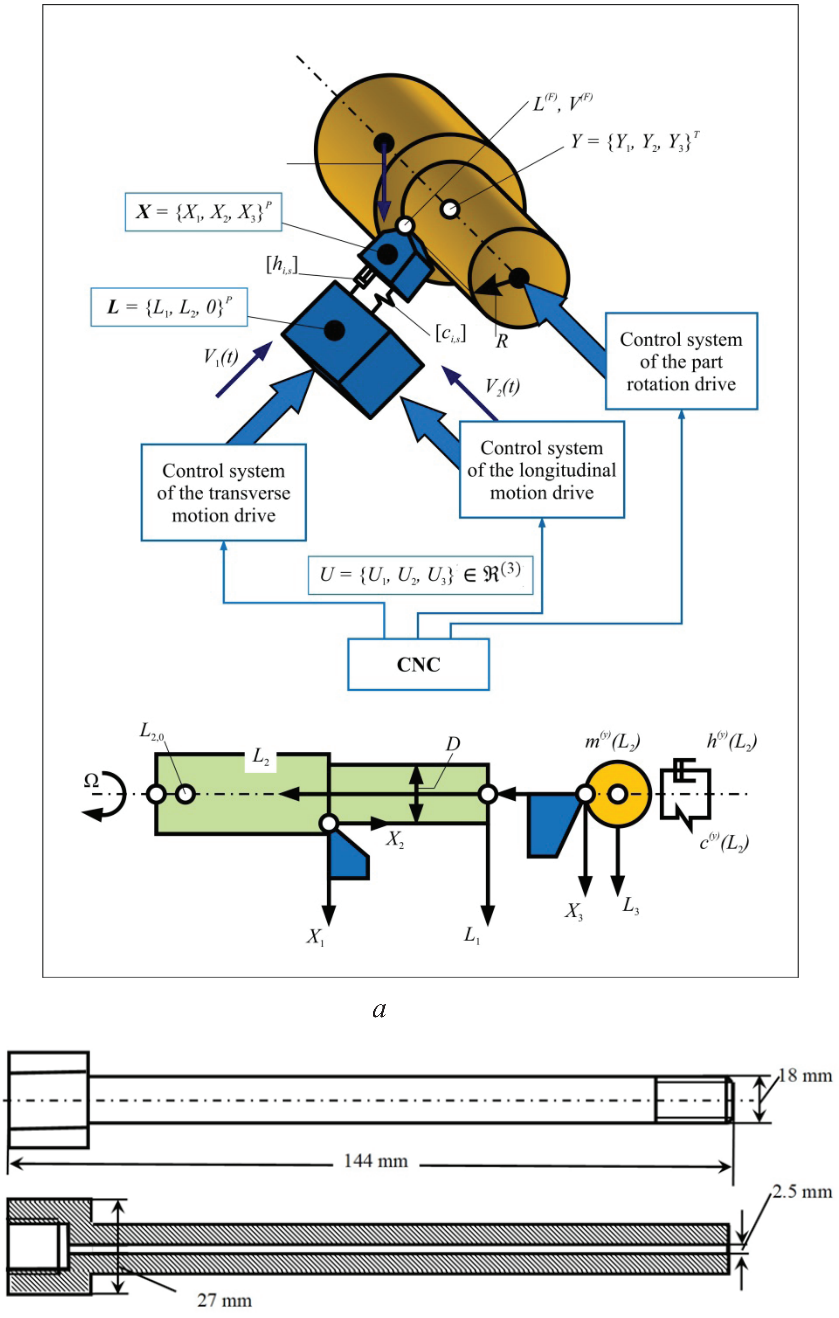
For citation: Zakovorotny V.L., Gvindjiliya V.E., Fesenko E.O. Application of the synergistic concept in determining the CNC program for turning. *Obrabotka metallov (tekhnologiya, oborudovanie, instrumenty) = Metal Working and Material Science*, 2022, vol. 24, no. 4, pp. 98–112. DOI: 10.17212/1994-6309-2022-24.4-98-112. (In Russian).

Introduction

Problem Statement. Many problems of the dynamics of technical systems began to be considered taking into account its system-synergetic analysis [3–7] after the publication of the works of *H. Haken* and *I. Prigozhin* [1, 2]. The system-synergetic approach has also been used to explain many phenomena in cutting and friction processing [8–10]. At the same time, in the last decade, the scientific community pays great attention to the development of a virtual model of machining processes on metal-cutting machines [11–30]. These models are intended for use at the stage of technological preparation of production of parts with a complex geometric profile. Complex shape parts are defined as those that require simultaneous changes in the longitudinal and transverse slide paths, as well as parts that change properties along the paths of the machine's executive elements (*TEE*).

*** Corresponding author**
Gvindjiliya Valery E., Post-graduate Student
Don State Technical University,
1 Gagarin square,
344000, Rostov-on-Don, Russian Federation
Tel.: +7 (918) 583-23-33, **e-mail:** sinedden@yandex.ru

The following steps are included in the basis of the structure of the synergistic processing control concept [3–7]. First, the control goal is formulated as the production of a batch of parts of a set quality while minimizing the reduced cost of its production. Secondly, the desired path $L_0^F = \left\{L_{1,0}^F, L_{2,0}^F, L_{3,0}^F\right\}^P \in \Re^{(3)}$ and the corresponding vector of velocities of forming motion $V_0^F = \frac{dL_0^F}{dt} = \left\{V_{1,0}^F, V_{2,0}^F, V_{3,0}^F\right\}^P \in \Re^{(3)}$ $\left(\text{and } \frac{dL^F}{dt} = V^F = \left\{V_1^F, V_2^F, V_3^F\right\}^P \in \Re^{(3)}\right)$ are determined, at which the requirements to the quality of parts are satisfied while minimizing the intensity of tool wear (Fig. 1, *a*).



b

Fig. 1. Controlled dynamic cutting system (*a*) and drawing of the “basic” part (*b*)

The space $\Re^{(3)}$ is defined by the directions of *TEE* motion, which are set by the *CNC* system. The shaping motion path is understood as the sum of the paths of the *TEE*, vector $/L = \{L_1, L_2, L_3\}^P \in \Re^{(3)}$, and the paths of the deformation displacements of the tool – vector $/X = \{X_1, X_2, X_3\}^P \in \Re^{(3)}$, and the workpiece – vector $/Y = \{Y_1, Y_2, Y_3\}^P \in \Re^{(3)}$ Consequently $L^{(F)} = L - X - Y$. The paths ***L*** and $V = \frac{dL}{dt} = \{Y_1, Y_2, Y_3\}^P \in \Re^{(3)}$ are defined by the *NC* program. The deformation displacements ***X*** and ***Y*** are considered in the motion coordinate system given by *TEE*. If ***X*** = 0, ***Y*** = 0, then $L^{(F)} = L$. The velocities $v^{(x)} = \frac{dX}{dt} = \{v_{X1}, v_{X2}, v_{X3}\}^P$ and $v^{(Y)} = \frac{dY}{dt} = \{v_{Y1}, v_{Y2}, v_{Y3}\}^P$ are also considered.

Thirdly, such coordination of the terminal path with the paths of the state space is provided, under which ***L***, ***X***, ***Y*** are asymptotically stable; – in this case $L_0^{\boldsymbol{F}}$ is an attractor. The difference between the synergistic paradigm of *NC* program synthesis and the traditional one is its definition based on the mutual agreement of all subsystems and the provision $L^{(F)} = L_0^{(F)}$ of the property of attraction of the entire state space. In addition, the conditions $L^{(F)} \in \aleph$, dictated by the quality requirements of the parts, should be met. And the dynamics of the system is taken into account as a whole.

Therefore, we rely on research in the field of cutting dynamics [24–43] to develop a synergistic approach to the machining processes control. There is a far from complete list of works on the dynamics of cutting. In spite of many works on the dynamics of cutting, it considers some particular models of the representation of cutting forces in the coordinates of an elastic system. The following ones are analyzed: buckling collapse conditions, formation of different attractors of deformation displacements of the tool and workpiece. It is necessary to analyze the dynamic system as a whole when solving the problem of synergistic synthesis, including determination of the desired path $L^{(F)} \in \aleph^{(F)}$ and corresponding paths $\boldsymbol{L}^{(F)}$, ***X*** and ***Y***. Here $\aleph^{(F)}$ – is the set of admissible variations of $\boldsymbol{L}^{(F)}$. The paper discusses all the stages of synergistic control of turning parts, the stiffness parameters of which change along the tool path: the methodology of creation $L_0^{(F)}$ and its asymptotic stability. An analysis of the effectiveness of synergistic control is presented on a specific example of manufacturing a "basic" part, the drawing of which is shown in Fig. 1, *b*.

Research methodology

Determining the desired path of the forming motion. When analyzing the system, it is advisable to use the principle of motions breaking down into "slow" ones, lying within the bandwidth of the motors of the machine's executive elements, and "fast" ones, determined by the dynamic properties of the tool and workpiece subsystems. Its use is based on the asymptotic properties of nonlinear differential equations with small parameters in the derivatives [44, 45]. Moreover, the subsystem of "fast" motions is considered in variations with respect to the paths of "slow" motions. If the subsystems are asymptotically stable, the path of the "slow" motions becomes an attractor. Typical for practice is the case when *TEE* are given and controllable within the servomotor bandwidths. Then we have the following equation of dynamics [40]:

$$\begin{cases} \mathbf{m}\dfrac{\mathbf{d^2X}}{\mathbf{dt^2}} + \mathbf{h}\dfrac{\mathbf{dX}}{\mathbf{dt}} + \mathbf{cX} = \mathbf{F(L, V, X, Y)}; \\ m^{(Y)}\dfrac{d^2Y_1}{dt^2} + h^{(Y)}\dfrac{dY_1}{dt} + c^{(Y)}Y_1 = F^{(0)}(\mathbf{L, V, X, Y})\chi_1, \end{cases} \tag{1}$$

where $\boldsymbol{m} = [m_s]$, $m_s = m$, $s = 1, 2, 3$ kgs^2/mm; $\boldsymbol{h} = [h_{s,l}]$, kgs^2/mm, $c = [c_{s,l}]$ kg/mm; $s,l = 1,2,3$ – are symmetric, positively determined matrices of inertial, velocity, and elastic coefficients. The subsystem of the workpiece has stiffness in the direction of its rotation axis by an order of magnitude greater than in other directions. In the plane Y_1–Y_2 it has full symmetry. Therefore, in this plane any orthogonal coordinate system is the main one. Then the force F_1 corresponds to the deformations only in the direction Y_1 [48]. The force $\boldsymbol{F}$ [4] can be represented in the form $\mathbf{F} = \{F_1, F_2, F_3\} = F^{(0)}\{\chi_1, \chi_2, \chi_3\}$. It is convenient to consider the following representation of modes (feed $S_P(p)$, depth $t_P(p)$ and cutting speed $V_P(p)$):

$$S_P(t) = \int_{t-T}^{t} \left[V_2(\xi) - v_{X_2}(\xi) - v_{Y_2}(\xi)\right] d\xi;\; V_P(t) =$$

$$= \pi D\Omega - v_{X_3}(t) - v_{Y_2}(t);\;\; t_P(t) = D/2 - (L_1 - X_1 - Y_1), \tag{2}$$

where $T = (\Omega)^{-1}$ is turnaround time, sec; D – diameter, m. If $\boldsymbol{X} = 0$, $\boldsymbol{Y} = 0$, then $S_P^{(0)}$, $t_P^{(0)}$, and $V_P^{(0)} = V_3 = \pi D\Omega$, are traditional modes. Then the model of relation of the force $F^{(0)}$ with the system coordinates has the equation [46, 47]:

$$T^{(0)} dF^{(0)}/dt + F^{(0)} = \rho\left\{1 + \mu\exp\left[-\varsigma(V_3 - v_{X_3})\right]\right\}\left[t_P^{(0)} - X_1 - Y_1\right] \int_{t-T}^{t} \left\{V_2(\xi) - v_{X_2}(\xi)\right\} d\xi, \tag{3}$$

where ρ – chip pressure on the leading edge of the tool, kg/mm^2; μ – dimensionless coefficient; ζ – steepness parameter of forces, sec·m^{-1}; $T^{(0)}$ – chip time constant, sec. If $\boldsymbol{L}$, $\boldsymbol{V}$ are set, then systems (1)–(3) allow to determine $\boldsymbol{X}$, $\boldsymbol{Y}$, and $\boldsymbol{L}^{(F)}$. If $L^{(\mathrm{F})} \in \aleph^{(\mathrm{F})}$, then $\boldsymbol{L}$, $\boldsymbol{V}$ determine *CNC* program. Otherwise, it is necessary to ensure the asymptotic stability of the resulting path and to choose $\boldsymbol{L}$, $\boldsymbol{V}$ or available variations of the parameters so that the condition $L^{(\mathrm{F})} \in \aleph^{(\mathrm{F})}$ or $L^{(\mathrm{F})} = L_0^{(\mathrm{F})}$ is satisfied.

To calculate $\boldsymbol{L}$, $\boldsymbol{V}$, at which $L^{(\mathrm{F})} \in \aleph^{(\mathrm{F})}$ is provided, let us use the principle of motions breaking down. First, a set of paths $L^{(\mathrm{F})} \in \aleph^{(\mathrm{F})}$ in "slow" time are defined; then in this set we choose those for which the trajectories of "fast" motions are asymptotically stable, and among them those for which the wear intensity is minimal. To determine the desired path of the "slow" motions, consider the values $\boldsymbol{L}$, $\boldsymbol{V}$ averaged over the period of rotation of the workpiece. For this purpose, (1) and (3) are considered in the slow discrete time $T = (\Omega) - 1$:

$$\boldsymbol{C}(iT)\,\boldsymbol{Z}(iT) = \boldsymbol{F}(iT) \tag{4}$$

where

$$\boldsymbol{Z}(iT) = \{X_1(iT), X_2(iT), X_3(iT), Y_1(iT)\}^T;\; \boldsymbol{F}(iT) = \rho_0 t_P^{(0)} S_P^{(0)}(iT)\{\chi_1, \chi_2, \chi_3\}^T;$$

$$\mathbf{C}(iT) = \begin{bmatrix} c_{1,1} + \rho_0 S_P^{(0)}(iT)\chi_1 & c_{2,1} & c_{3,1} & \rho_0 S_P^{(0)}(iT)\chi_1 \\ c_{1,2} + \rho_0 S_P^{(0)}(iT)\chi_2 & c_{2,2} & c_{3,2} & \rho_0 S_P^{(0)}(iT)\chi_2 \\ c_{1,3} + \rho_0 S_P^{(0)}(iT)\chi_3 & c_{2,3} & c_{3,3} & \rho_0 S_P^{(0)}(iT)\chi_3 \\ \rho_0 S_P^{(0)}(iT)\chi_1 & 0 & 0 & c^{(Y)} + \rho_0 S_P^{(0)}(iT)\chi_1 \end{bmatrix},\;\; \rho_0 = \rho\left[1 + \mu\exp(-\varsigma V_3)\right]$$

In (4) $\sum_{i=1}^{i=s} iT = L_0$ (Fig. 1). From (4) $0.5\Delta D(iT) = X_1(iT) + Y_1(iT)$ is calculated. During processing it is required to ensure the condition $\Delta D(iT) = \text{const}$ [51–55]. Let us determine $\Delta D(iT)$ from (4):

$$\Delta D(iT) = 2t_P^{(0)} S_P^{(0)} \frac{\Delta_X + \Delta_Y}{\Delta} \qquad (5)$$

where

$$\Delta(iT) = \begin{bmatrix} c_{1,1} + \rho_0 S_P^{(0)}(iT)\chi_1 & c_{2,1} & c_{3,1} & \rho_0 S_P^{(0)}(iT)\chi_1 \\ c_{1,2} + \rho_0 S_P^{(0)}(iT)\chi_2 & c_{2,2} & c_{3,2} & \rho_0 S_P^{(0)}(iT)\chi_2 \\ c_{1,3} + \rho_0 S_P^{(0)}(iT)\chi_3 & c_{2,3} & c_{3,3} & \rho_0 S_P^{(0)}(iT)\chi_3 \\ \rho_0 S_P^{(0)}(iT)\chi_1 & 0 & 0 & c^{(Y)} + \rho_0 S_P^{(0)}(iT)\chi_1 \end{bmatrix};$$

$$\Delta_X(iT) = \begin{bmatrix} \chi_1 & c_{2,1} & c_{3,1} & \rho_0 S_P^{(0)}(iT)\chi_1 \\ \chi_2 & c_{2,2} & c_{3,2} & \rho_0 S_P^{(0)}(iT)\chi_2 \\ \chi_3 & c_{2,3} & c_{3,3} & \rho_0 S_P^{(0)}(iT)\chi_3 \\ \chi_1 & 0 & 0 & c^{(Y)} + \rho_0 S_P^{(0)}(iT)\chi_1 \end{bmatrix};$$

$$\Delta_Y(iT) = \begin{bmatrix} c_{1,1} + \rho_0 S_P^{(0)}(iT)\chi_1 & c_{2,1} & c_{3,1} & \chi_1 \\ c_{1,2} + \rho_0 S_P^{(0)}(iT)\chi_2 & c_{2,2} & c_{3,2} & \chi_2 \\ c_{1,3} + \rho_0 S_P^{(0)}(iT)\chi_3 & c_{2,3} & c_{3,3} & \chi_3 \\ \rho_0 S_P^{(0)}(iT)\chi_1 & 0 & 0 & \chi_1 \end{bmatrix}.$$

An analysis of equation (5) shows that, depending on $\Delta D(iT)$, there is a branching of solutions $S_P^{(0)}(iT)$. Moreover, there are areas in which there are no real solutions. It all depends on the ratio of stiffness of the workpiece subsystem $c^{(Y)}$, matrix elements c and allowable values $\Delta D(iT)$ for given values of the allowance $t_P^{(0)}$. In the case if $c \to \infty$ and $\Delta D(iT) = \Delta D_0 = \text{const.}$, then (5) becomes an equation:

$$\Delta D_0 = 2t_P^{(0)} \frac{\rho_0(V_P) S_P^{(0)}(iT)\chi_1}{c^{(Y)}(iT) + \rho_0(V_P) S_P^{(0)}(iT)\chi_1}, \qquad (6)$$

from which the set of paths $S_P^{(0)}(iT,V_P)$ is determined, each of which depends on the cutting speed $V_P = \text{const}$. Obviously:

$$S_P^{(0)}(iT,V_P) = \frac{\Delta D_0 c^{(Y)}(iT)}{\rho_0(V_P)\chi_1[t_P^{(0)} - \Delta D_0]}, \qquad (7)$$

$S_P^{(0)}(iT,V_P)$ – path of the feed per revolution along the slide motion, at which $\Delta D_0 = \text{const}$.

Analysis of the equations (5) and (6) makes it possible to draw some conclusions about the choice of tool parameters and cutting modes to reduce the influence of elastic deformations on the part diameter, also known from practice.

1. As the depth of cut decreases, the diameter variations due to changes in stiffness decrease. Therefore, machining a part, the stiffness of which changes along the tool path, is performed in several passes with a successive decrease in the allowance.

2. The value $\Delta D(iT)$ is influenced by the angular coefficient χ_1. Angular coefficients depend on the tool geometry, first of all, on the back rake angle and the rake angle [56].

3. To reduce the uncertainty $\Delta D(iT)$ for variations $\boldsymbol{c}$, it is reasonable to carry out machining with small feeds. However, at small feeds commensurate with the tool radius at its apex, the stabilizing effect of the motion direction formed on the workpiece is leveled out. Therefore, the magnitude of the feed from below is also limited [57]. These methods do not eliminate the need for matching the *CNC* program with the law of change of stiffness.

If the set of paths $S_P^{(0)}(iT,V_P)$ is calculated, then it is additionally necessary to select from this set those, for which the condition of minimum wear intensity is satisfied. The solution to this problem is described in sufficient detail in [58]. The paths (7) are calculated under the assumption that the subsystem of "fast" motions is asymptotically stable. Under this condition (7), there is an attractor that has the property of attraction in the entire state space. In this connection, it is additionally necessary to analyze the subsystem of "fast" motions for asymptotic stability.

Example of CNC program matching with a change in part stiffness.

The problem of matching *TEE* with changing system properties has a wide range of applications: matching *TEE* with evolutionary changes in system properties due to the work of forces in the cutting zone; matching *TEE* with an a priori specified law of changing the stiffness of the workpiece; matching *TEE* with the development of tool wear, etc. As an example, the longitudinal turning of the fuel pump nozzle fitting of a diesel engine is considered (length L_0 = 144 mm (Fig. 1, *b*), diameter D = 18 mm, material – hot-rolled bar made of *steel 45* (STATE STANDARD 2590-2006) with diameter D = 25 mm. The tools used were tool systems with interchangeable 15 % (*WC* + *TiC*) + 6 % *Co* (*HS123*) square inserts and *MR TNR 2020 K11* toolholders. Tool geometry: back rake angle $\gamma = 15°$, cutting edge angle $\varphi = 90°$, front clearance angle $\alpha = 6^{o}$. Parameters of the tool elastic system and dynamic coupling are given in Table 1, Table 2. Generalized mass $m = 0.5 \cdot 10^{-3}$ kg·s^2/mm. To determine the law of variation of the radial stiffness along the axis of the workpiece, we can use the laws of bending vibrations of rods [59]. This information is easier and more accurate to obtain experimentally (Fig. 1, *a*). The law $c^{(Y)}(L_2)$ should be supplemented by its agreement with the change in the reduced mass along L_2. This is due to the fact that the natural frequencies of the bending vibrations of the shaft remain constant at all values of L_2 [4–6, 46].

Table 1

Matrices of velocity coefficients and elasticity of the tool subsystem

$c_{1,1}$, kg/mm	$c_{2,2}$, kg/mm	$c_{3,3}$, kg/mm	$h_{1,1}$, kg·s/mm	$h_{2,2}$, kg·s/mm	$h_{3,3}$, kg·s/mm
2,000	1,000	1,000	1.3	1.1	0.8
$c_{1,2} = c_{2,1}$, кг/мм	$c_{1,3} = c_{3,1}$, кг/мм	$c_{2,3} = c_{3,2}$, кг/мм	$h_{1,2} = h_{2,1}$, кг · с/мм	$h_{1,3} = h_{3,1}$, кг · с/мм	$h_{2,3} = h_{3,2}$, кг · с/мм
$c_{1,2} = c_{2,1}$, kg/mm	$c_{1,3} = c_{3,1}$, kg/mm	$c_{2,3} = c_{3,2}$, kg/mm	$h_{1,2} = h_{2,1}$, kg·s/mm	$h_{1,3} = h_{3,1}$, kg·s/mm	$h_{2,3} = h_{3,2}$, kg·s/mm
100	150	80	0.6	0.5	0.4

Table 2

Dynamic link options

ρ, kg/mm^2	ζ, (mm/s)$^{-1}$	$T^{(0)}$, s	μ	χ_1	χ_2	χ_3
300	0.1	0.0001–0.0005	0.5	0.7	0.5	0.5

The path $S_P^{(0)}(L_2)$ in Fig. 2, *b* corresponds to the speed of longitudinal motions of the slide $V_2L_2 = S_P^{(0)}(L_2)(T)^{-1}$. The paths shown in Fig. 2, *c* characterize the attracting set of deformational displacement along the whole path of the tool motion in the case when the subsystem of "fast" motions is asymptotically stable. The paths in Fig. 2 are obtained by assuming that $V_3 = 1.5$ m/s $= V_P =$ const. Varying V_3 and $t_P^{(0)}$ will lead to a displacement of the path (Fig. 2).

Let us consider the problem of asymptotic stability of deformational displacements for the subsystem of "fast" motions. Quasi-permanent paths of deformational displacements $X_s^{(*)}(iT), s = 1,2,3$; $Y^{(*)}(iT)$; force $F^{(*,0)}(iT)$ and velocities $V_2^{(*)}(iT)$ correspond to the curves (Fig. 2). $X_s^{(*)}(iT)$, $Y^{(*)}(iT)$, and $F^{(*,0)}(iT)$ are slowly changing state coordinates. Equation in variations with respect to the paths of "slow" motions is obtained after replacing of $X_s(t) - X_s^{(*)}(iT) = x_s(t)$; $Y(t) - Y^{(*)}(iT) = y(t)$; and $F^{(0)}(t) - F^{(0,*)}(iT) = f(t)$;

Its linearization in the vicinity, $X_s^{(*)}(iT), s = 1,2,3$; $Y^{(*)}(iT)$; and $F^{(*,0)}(iT)$ leads to a system of linear equations with lagged arguments. The analysis of stability of such systems on the basis of algebraic criteria, as well as the *Mikhailov* criterion, is not fair [47, 48]. The simulation of forces in state coordinates allows us to interpret the forces as feedbacks in the system. Therefore, *Nyquist* stability criteria are used, for which

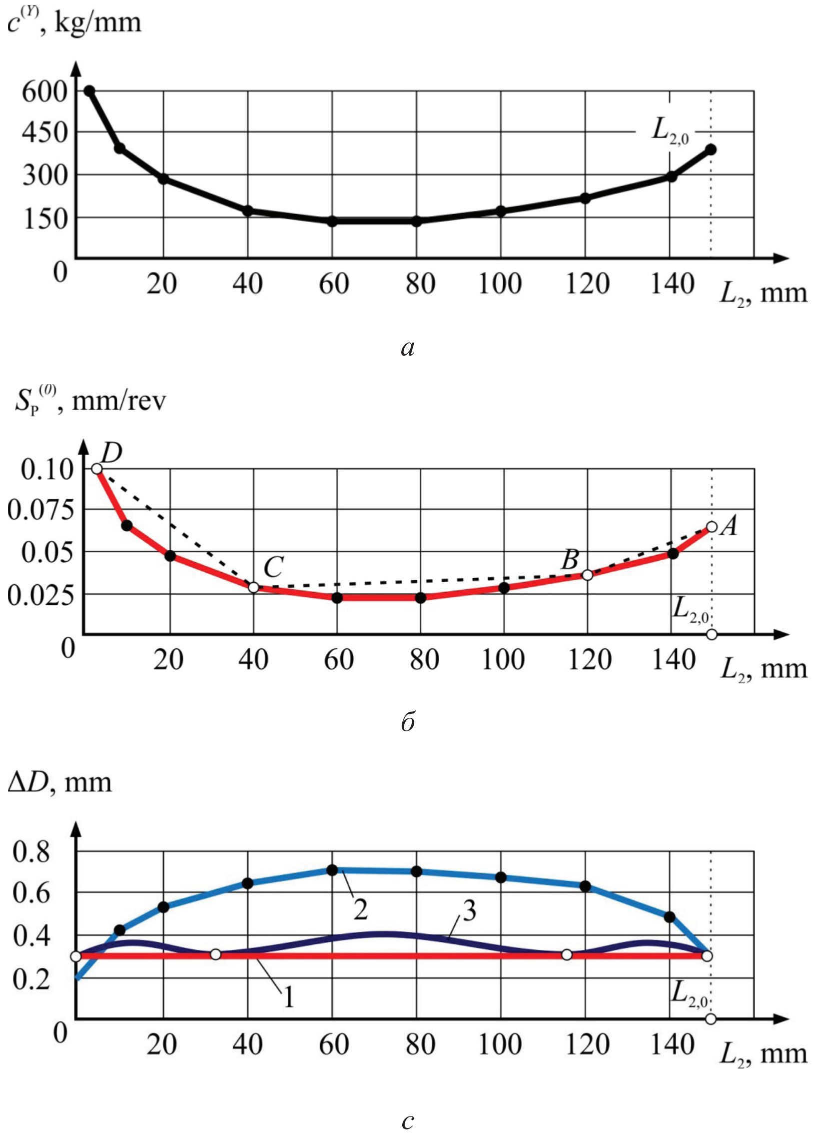


Fig. 2. An example of matching the path of changing the radial stiffness of the workpiece (*a*) with the value of the feed per revolution (*b*), changing the diameter variations (*c*):

1 – complete matching along the path; *2* – processing with a constant feed;
3 – linear interpolation of feed change over four nodal points (*A-B-C-D*)

from (1) taking into account (3) the transfer function $W_P(p)$ of the system in the open state is obtained for the linearized system in variations:

$$W_P(p) = \frac{\rho_0}{T^{(0)}p+1}\Big\{S_P^{(0)} g_{FX_1}(p)\left[1-\exp(-Tp)\right] +$$

$$+\ t_P^{(0)} g_{FX_2}(p)\left[1-\exp(-Tp)\right] + S_P^{(0)} g_{FY}(p)\left[1-\exp(-Tp)\right]\Big\}, \quad (8)$$

where

$$g_{FX_1}(p) = \Delta_{X_1} / \Delta;\ g_{FX_2}(p) = \Delta_{X_2} / \Delta;\ g_{FY}(p) = \Delta_Y / \Delta;$$

$$\Delta = \begin{bmatrix} c_{1,1} & c_{2,1} & c_{3,1} & 0 \\ c_{1,2} & c_{2,2} & c_{3,2} & 0 \\ c_{1,3} & c_{2,3} & c_{3,3} & 0 \\ 0 & 0 & 0 & c^{(Y)} \end{bmatrix}; \quad \Delta_{X_1} = \begin{bmatrix} \chi_1 & c_{2,1} & c_{3,1} & 0 \\ \chi_2 & c_{2,2} & c_{3,2} & 0 \\ \chi_3 & c_{2,3} & c_{3,3} & 0 \\ \chi_1 & 0 & 0 & c^{(Y)} \end{bmatrix};$$

$$\Delta_{X_2} = \begin{bmatrix} c_{1,1} & \chi_1 & c_{3,1} & 0 \\ c_{1,2} & \chi_2 & c_{3,2} & 0 \\ c_{1,3} & \chi_3 & c_{3,3} & 0 \\ 0 & \chi_1 & 0 & c^{(Y)} \end{bmatrix}; \quad \Delta = \begin{bmatrix} c_{1,1} & c_{2,1} & c_{3,1} & \chi_1 \\ c_{1,2} & c_{2,2} & c_{3,2} & \chi_2 \\ c_{1,3} & c_{2,3} & c_{3,3} & \chi_3 \\ 0 & 0 & 0 & \chi_1 \end{bmatrix}.$$

The equation (8) includes technological parameters $t_P^{(0)}$ and $S_P^{(0)}$, and indirectly V_P, because $T^{(0)}$ mainly depends on the cutting speed, and spindle speed. Parameters $t_P^{(0)}$, $S_P^{(0)}$ and Ω are defined by the *CNC* program. Here is an example of changing the stability area in the plane of two varying parameters "$V_P - \rho$" (Fig. 3). Above the figurative lines, the system is unstable. The curves are typical when evaluating the effect of spindle speed on the stability of elastic deformational displacements of the tool and workpiece. Periodic bursts of admissible values ρ explained by the regenerative effect of the integral feed forming operator. Analysis of the properties of the system at $t < T$ shows that the cutting speed primarily changes the parameter $T^{(0)}$. Its increase causes two opposite trends. On the one hand, an increase of $T^{(0)}$ leads to additional phase shifts between forces and strains, which contributes to the loss of stability. On the other hand, an increase of $T^{(0)}$ contributes to an increase in the damping effect of the cutting process on oscillations, which stabilizes the equilibrium. Therefore, there is a minimum when V_P is increased.

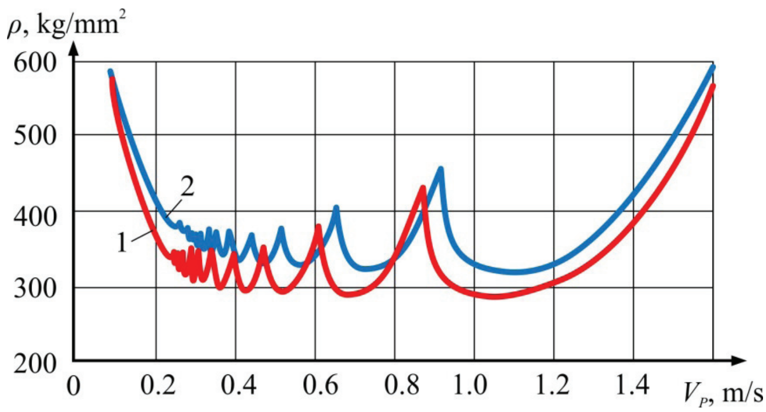


Fig. 3. An example of changing the region of stability in a plane "$V_P - \rho$": $1 - c^{(Y)} = 400$ kg/mm; $2 - c^{(Y)} = 150$ kg/mm

If we follow the work [58], as the cutting speed increases, a redistribution of physical interactions in the interface areas of the tool and cutting zone (adhesion, fatigue, abrasion, tribochemical, diffusion, etc.) is observed. Transition from prevailing adhesion to diffusion interactions corresponds to the minimum of wear intensity. This range corresponds to the minimum power of irreversible transformations of the energy input to the cutting [56]. When machining *steel 45*, the region of this transition is located in the range 1.0–2.0 m/s.

In addition, with increasing Ω there is a critical value of Ω, starting from which parametric effects begin to appear, including parametric self-excitation [45]. Therefore, there is a critical value of Ω, exceeding which is unacceptable and it is necessary to take into account the restrictions imposed on the system the peculiarities of its dynamics.

Results and discussion

One of the currently developing ways to increase the efficiency of machining parts, the stiffness of which varies along the tool path is the coordination of external control (from the *CNC* system) with internal control of elastic deformations, determined by the dependence of forces on deformation displacements and *TEE*. The synergistic problem of interaction between the external control and the internal control [3–7], formed by the dependence of cutting forces on deformation displacements and *TEE* set by the *CNC* system, is solved on the basis of matching the external control with the internal dynamics of cutting. To do this, firstly, when defining the *CNC* program, a set of desired paths of the forming motions, which include both the *TEE* and the deformations of the tool tip relative to the workpiece, is constructed. The desired paths are defined on the basis of the requirements to the parts quality. All intermediate paths of the dynamical system up to the *CNC* program are subject to these paths. Thus, the currently existing principle of subordination is replaced by the principle of interaction of subsystems according to the direction of goal achievement. Secondly, from the obtained set of paths obtained at different technological modes, a path is selected, at which the intensity of tool wear is minimal. The depth of cutting is usually set a priori, so when selecting the modes, it is necessary to determine the cutting speed, at which the desired path of the forming motions is asymptotically stable, from the conditions of physical optimality. The performed verification of the effectiveness of the method in the longitudinal turning of the fuel pump nozzle showed that by reducing the number of passes, it is possible to increase the productivity of part manufacturing up to two times in machine time. The increase in productivity is achieved, firstly, by reducing the number of passes. According to the basic technology, after drilling the central hole $D = 2.5$ mm, which is performed on a specialized machine, the workpiece is installed in the centers and the longitudinal turning of the shaft is performed in the beginning for the entire length up to $D = 27$ mm. Then the main surface of the shaft is turned in three passes with depths of cut $t_P^{(0)} = 1.5$ mm; $t_P^{(0)} = 0.7$ mm; and $t_P^{(0)} = 0.3$ mm. Such transitions provide the required diameter accuracy until the back edge wear reaches 0.6 mm. The cutting modes remain unchanged. When using the synergistic concept due to the coordination of *TEE* with the changing parameters of the workpiece stiffness along the tool motion coordinates, coordination of *TEE* with the evolutionary changes in the properties of the dynamic relationship formed by cutting, as well as determining the optimal coordinates of switching (readjustment) of machining cycles, it is possible to reduce the number of passes from four to two. Besides, it is possible to increase the batch of parts before tooling system readjustment by 1.5 times. The optimal switching coordinates were determined according to the method stated by us earlier [59]. It is important to emphasize that the increase in productivity was achieved by software methods, without additional material costs.

Conclusion

One of the promising directions of manufacturing a batch of parts of a set quality while minimizing the reduced costs is the use of the synergistic principle of matching the external control (*CNC* program) with the internal dynamics of the system. The given example of manufacturing a part, the stiffness of which clearly depends on the tool path, allows increasing the productivity in terms of machine time by up to two times. The developed approach and the given algorithms for determining the desired path of the forming motions and the corresponding *NC* program on the basis of synergetic matching of the external control with the changing dynamic characteristics of the workpiece can be extended to a large class of parts with a complex geometric shape.

References

1. Prigogine I., Stengers I. *Order out of chaos*. London, Heinemann, 1984 (Russ. ed.: Prigozhin I., Stengers I. *Poryadok iz khaosa*. Moscow, Progress Publ., 1986. 432 p.).

2. Haken H. *Advanced synergetics: instability hierarchies of self-organizing systems and devices*. Berlin, New York, Springer, 1983 (Russ. ed.: Khaken G. *Sinergetika: ierarkhii neustoichivostei v samoorganizuyushchikhsya sistemakh i ustroistvakh*. Moscow, Mir Publ., 1985. 424 p.).

3. Kolesnikov A.A., ed. *Sinergetika i problemy teorii upravleniya* [Synergetics and problems in control theory]. Moscow, Fizmatlit, 2004. 504 p. ISBN 5-9221-0336-9.

4. Zakovorotny V.L., Flek M.B. *Dinamika protsessa rezaniya. Sinergeticheskii podkhod* [The dynamics of the cutting process. Synergistic approach]. Rostov-on-Don, Terra Publ., 2005. 880 p.

5. Zakovorotny V.L., Gvindjiliya V.E. Sinergeticheskaya kontseptsiya pri programmnom upravlenii protsessami obrabotki na metallorezhushchikh stankakh [Synergetic concept of software control of machining processes on metal-cutting machines]. *Izvestiya vysshikh uchebnykh zavedenii. Mashinostroenie = BMSTU Journal of Mechanical Engineering*, 2021, no. 5 (734), pp. 24–36. DOI: 10.18698/0536-1044-2021-5-24-36.

6. Zakovorotny V.L., Gvindjiliya V.E. Sinergeticheskii podkhod k povysheniyu effektivnosti upravleniya protsessami obrabotki na metallorezhushchikh stankakh [Synergetic approach to improve the efficiency of machining process control on metal-cutting machines]. *Obrabotka metallov (tekhnologiya, oborudovanie, instrumenty) = Metal Working and Material Science*, 2021, vol. 23, no. 3, pp. 84–99. DOI: 10.17212/1994-6309-2021-23.3-84-99.

7. Zakovorotny V., Gvindjiliya V. Process control synergetics for metal-cutting machines. *Journal of Vibration Engineering*, 2022, vol. 24 (1), pp. 177–189. DOI: 10.21595/jve.2021.22087.

8. Zakovorotny V.L. *Nelineinaya tribomekhanika* [Nonlinear tribomechanics]. Rostov-on-Don, Don State Technical University Publ., 2000. 293 p.

9. Ryzhkin A.A. *Sinergetika iznashivaniya instrumental'nykh materialov pri lezviinoi obrabotke* [Synergetics of tool wear in cutting edge treatment]. Rostov-on-Don, Don State Technical University Publ., 2019. 289 p. ISBN 978-5-7890-1669-5.

10. Kabaldin Yu.G., Shatagin D.A. Artificial intelligence and cyberphysical machining systems in digital production. *Russian Engineering Research*, 2020, vol. 40, no. 4, pp. 292–296. DOI: 10.3103/S1068798X20040115. Translated from *Vestnik mashinostroeniya*, 2020, no. 1, pp. 21–25.

11. Altintas Y., Kersting P., Biermann D., Budak E., Denkena B. Virtual technological systems for parts processing operations. *CIRP Annals*, 2014, vol. 63 (2), pp. 585–605. DOI: 10.1016/j.cirp.2014.05.007.

12. Arrazola P., Ozel T., Umbrello D., Davies M., Jawahir I. Recent advances in modelling of metal machining processes. *CIRP Annals*, 2013, vol. 62 (2), pp. 695–718. DOI: 10.1016/j.cirp.2013.05.006.

13. Pantyukhin O.V., Vasin S.A. Tsifrovoi dvoinik tekhnologicheskogo protsessa izgotovleniya izdelii spetsial'nogo naznacheniya [Digital double of the technological process of manufacturing special-purpose products]. *Stankoinstrument*, 2021, no. 1 (22), pp. 56–59. DOI: 10.1016/j.cirp.2013.05.006. (In Russian).

14. Altintas Y., Brecher C., Weck M., Witt S. Virtual machine tool. *CIRP Annals*, 2005, vol. 54 (2), pp. 115–138. DOI: 10.1016/S0007-8506(07)60022-5.

15. Erkorkmaz K., Altintas Y., Yeung C.-H. Virtual computer numerical control system. *CIRP Annals*, 2006, vol. 55 (1), pp. 399–402. DOI: 10.1016/S0007-8506(07)60444-2.

16. Ahamed As., Ahamed At., Katuwawala D., Ee T.T., Tan Z.H., Bajaj I.S., Wickramasurendra Th., Namazi H. Complexity-based analysis of the influence of machining parameters on the surface finish of drilled holes in drilling operation. *Fractals*, 2019, vol. 27 (6), p. 1950087. DOI: 10.1142/S0218348X19500877.

17. Kilic Z.M., Altintas Y. Generalized mechanics and dynamics of metal cutting operations for unified modeling. *International Journal of Machine Tools and Manufacture*, 2016, vol. 104, pp. 1–13. DOI: 10.1016/j.ijmachtools.2016.01.006.

18. Estman L., Merdol D., Brask K.-G., Kalhori V., Altintas Y. Developing strategies for machining aerospace components using virtual machining tools. *New Production Technologies in Aerospace Industry.* Cham, Springer, 2014, pp. 63–68. DOI: 10.1007/978-3-319-01964-2_9.

19. Yangui H., Zghal B., Kessentini A., Chevallier G., Rivière A., Haddar M., Karra Ch. Influence of cutting and geometrical parameters on the cutting force in milling. *Engineering*, 2010, vol. 2 (10), pp. 751–761. DOI: 10.4236/eng.2010.210097.

20. Thasana W., Chianrabutra S. A comparison between simulation and experiment of virtual machining in CNC turning machine considering kinematic motion deviations, tool wear and workpiece deflection errors. *Journal of Advanced Mechanical Design, Systems and Manufacturing*, 2019, vol. 13 (1), p. 18-00250. DOI: 10.1299/jamdsm.2019jamdsm0009.

21. Thasana W., Sugimura N., Iwamura K., Tanimizu Y. A study on estimation of three-dimensional tolerances based on simulation of virtual machining in turning processes including kinematic motion deviations. *Journal of Advanced Mechanical Design*, *Systems, and Manufacturing*, 2015, vol. 9 (1), p. 14-00507. DOI: 10.1299/jamdsm.2015jamdsm0012.

22. Studiyanti L. Workstation and posture improvement in cutting machine process using virtual modeling. *IOP Conference Series: Materials Science and Engineering*, 2021, vol. 1072, p. 012032. DOI: 10.1088/1757-899X/1072/1/012032.

23. Soori M., Arezoo B. Virtual machining systems for CNC milling and turning machine tools: a review. *International Journal of Engineering and Technology*, 2020, vol. 18, pp. 56–104.

24. Liu H., Xu X., Zhang J., Liu Z., He Y., Zhao W., Liu Z.-q. The state of the art for numerical simulations of the effect of the microstructure and its evolution in the metal-cutting processes. *International Journal of Machine Tools and Manufacture*, 2022, vol. 177, p. 103890. DOI: 10.1016/j.ijmachtools.2022.103890.

25. Beri B., Miklos A., Takacs D., Stepan G. Nonlinearities of hardware-in-the-loop environment affecting turning process emulation. *International Journal of Machine Tools and Manufacture*, 2020, vol. 157, p. 103611. DOI: 10.1016/j.ijmachtools.2020.103611.

26. Soori M., Arezoo B., Habibi M. Virtual machining with dimensional, geometric and tool deflection errors on three-axis CNC milling machines. *Journal of Manufacturing Systems*, 2014, vol. 33 (4), pp. 498–507. DOI: 10.1016/j.jmsy.2014.04.007.

27. Duvedi R.K., Bedi S., Batish A., Mann S. Multi-point method for 5-axis processing of triangular surface models. *Computer-Aided Design*, 2014, vol. 52, pp. 17–26. DOI: 10.1016/j.cad.2014.02.008.

28. Gan V.F., Fu J.Z., Shen H.Yu., Chen Z.Yu., Lin Z.V. Creating a 5-axis toolpath in CNC machining T-spline surfaces. *Computer-Aided Design*, 2014, vol. 52, pp. 51–63. DOI: 10.1016/j.cad.2014.02.013.

29. Kiswanto G., Hendrikoh H., Duk E. An analytical method for obtaining cutter engagement with the workpiece during semi-finishing in five-axis milling. *Computer-Aided Design*, 2014, vol. 55, pp. 81–93. DOI: 10.1016/j.cad.2014.05.003.

30. Wu D., Rosen D.W., Wang L., Schaefer D. Cloud design and manufacturing: a new paradigm in digital manufacturing and design innovation. *Computer-Aided Design*, 2015, vol. 59, pp. 1–14. DOI: 10.1016/j.cad.2014.07.006.

31. Kudinov V.A. *Dinamika stankov* [Dynamics of machine tools]. Moscow, Mashinostroenie Publ., 1967. 359 p.

32. Tlusty J., Polacek A., Danek C., Spacek J. *Selbsterregte Schwingungen an Werkzeugmaschinen*. Berlin, VerlagTechnik, 1962. 320 p.

33. Merritt H.E. The theory of the contribution of self-excited chatter of machine tools to the study of vibrations of machine tools. *ASME Journal of Engineering for Industry*, 1965, vol. 87 (4), pp. 447–454. DOI: 10.1115/1.3670861.

34. Altintas Y. Analytical prediction of 3D vibration in milling. *JSME International Journal. Mechanical Systems, Machine Elements and Manufacturing,* 2001, vol. 44 (3), pp. 717–723. DOI: 10.1299/jsmec.44.717.

35. Stépán G., Insperger T., Salai R. Delay, parametric excitation and nonlinear dynamics of cutting processes. *International Journal of Bifurcation and Chaos*, 2005, vol. 15 (9), pp. 2783–2798. DOI: 10.1142/S0218127405013642.

36. Stépán G. Modeling of non-linear regenerative effects in metal cutting. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 2001, vol. 359 (1781), pp. 739–757. DOI: 10.1098/rsta.2000.0753.

37. Reith M.J., Bachrathy D., Stepan G. Improving the stability of multi-cutter turning with detuned dynamics. *Machining Science and Technology*, 2016, vol. 20 (3), pp. 440–459. DOI: 10.1080/10910344.2016.1191029.

38. Voronov S.A., Kiselev I.A. Nelineinye zadachi dinamiki protsessov rezaniya [Nonlinear problems of cutting process dynamics]. *Mashinostroenie i inzhenernoe obrazovanie = Mechanical Engineering and Engineering Education*, 2017, no. 2 (51), pp. 9–23.

39. Rusinek R., Wiercigroch M., Wahi P. Influence of tool flank forces on complex dynamics of a cutting process. *International Journal of Bifurcation and Chaos*, 2014, vol. 24 (9), pp. 189–201. DOI: 10.1142/S0218127414501156.

40. Rusinek R., Wiercigroch M., Wahi P. Modeling of frictional chatter in metal cutting. *International Journal of Mechanical Sciences*, 2014, vol. 89, pp. 167–176. DOI: /10.1016/j.ijmecsci.2014.08.020.

41. Gouskov A., Gouskov M., Lorong Ph., Panovko G. Influence of the clearance face on the condition of chatter self-excitation during turning. *International Journal of Machining and Machinability of Materials*, 2017, vol. 19 (1), pp. 17–39. DOI: 10.1504/IJMMM.2017.10002088.

42. Guskov M., Dyn Dyk T., Panovko G., Guskov A.M. Modelirovanie i issledovanie ustoichivosti protsessa mnogoreztsovogo rezaniya «po sledu» [Modeling and investigation of the stability of the process of multi-cutting cutting "on the trail"]. *Problemy mashinostroeniya i nadezhnosti mashin*, 2018, no. 3, pp. 19–27. DOI: 10.31857/S023571190000533-7. (In Russian).

43. Pontryagin L.S. *Izbrannye nauchnye trudy*. T 2 [Selected works. Vol. 2]. Moscow, Nauka Publ., 1988, pp. 95–154.

44. Tikhonov A.N. Sistemy differentsial'nykh uravnenii, soderzhashchie malye parametry pri proizvodnykh [Systems of differential equations containing small parameters with derivatives]. *Matematicheskii sbornik*, 1952, vol. 31, no. 3, pp. 575–586. (In Russian).

45. Zakovorotniy V.L., Pham D.T., Nguyen X.C., Ryzhkin M.N. Modelirovanie dinamicheskoi svyazi, formiruemoi protsessom tocheniya, v zadachakh dinamiki protsessa rezaniya (pozitsionnaya svyaz') [Dynamic coupling modeling formed by turning in cutting dynamics problems (positional coupling)]. *Vestnik Donskogo gosudarstvennogo tekhnicheskogo universiteta = Vestnik of Don State Technical University*, 2011, vol. 11, no. 3 (54), pp. 301–311.

46. Loitsyanskii L.G. *Kurs teoreticheskoi mekhaniki* [The course of theoretical mechanics]. Moscow, Nauka Publ., 1982. 640 p.

47. Lyapunov A.M. *Obshchaya zadacha ob ustoichivosti dvizheniya* [The general problem of motion stability]. St. Petersburg, Gostekhizdat Publ., 1950. 472 p.

48. Merkin D.R. *Vvedenie v teoriyu ustoichivosti dvizheniya* [Introduction to the theory of motion stability]. Moscow, Nauka Publ., 1971. 312 p.

49. Brzhozovskiy B.M., Yankin I.N., Khayrov D.A. Obespechenie kachestva rezaniya neodnorodnykh materialov na osnove optimal'noi dinamicheskoi nastroiki formoobrazuyushchikh podsistem stanka [Providing of quality of cutting of heterogeneous materials on basis of optimum dynamic tuning of shape-generating subsystems of machine-tool]. *Progresivni tekhnologiї i sistemi mashinobuduvannya = Progressive Technologies and Machine-Building Systems*, 2012, no. 1 (43), pp. 71–77.

50. Balakshin B.S., ed. *Adaptivnoe upravlenie stankami* [Adaptive control of machine tools]. Moscow, Mashinostroenie Publ., 1973. 688 p.

51. Bazrov B.M. *Tekhnologicheskie osnovy proektirovaniya samopodnastraivayushchikhsya stankov* [Technological bases for designing self-adjusting machine tools]. Moscow, Mashinostroenie Publ., 1978. 216 p.

52. Tverskoi M.M. *Avtomaticheskoe upravlenie rezhimami obrabotki na stankakh* [Automatic control of processing modes on machines]. Moscow, Mashinostroenie Publ., 1982. 208 p.

53. Veits V.L., Vasil'kov D.V. Zadachi dinamiki, modelirovaniya i obespecheniya kachestva pri mekhanicheskoi obrabotke malozhestkikh zagotovok [Dynamics, modeling, and quality maintenance at the machine treatment of low-rigidity billets]. *Stanki i instrument = Machines and Tools*, 1999, no. 6, pp. 9–13.

54. Zakovorotny V.L., Gvindzhiliya V.E., Minakov V.S. Vliyanie geometrii rezhushchego instrumenta na dinamiku protsessa tocheniya [Cutting-tool geometry effect on turning process dynamics]. *Vestnik Donskogo gosudarstvennogo tekhnicheskogo universiteta = Vestnik of Don State Technical University*, 2018, vol. 18, no. 2, pp. 201–213. DOI: 10.23947/1992-5980-2018-18-2-201-213.

55. Zakovorotny V.L., Gvindzhiliya V.E. Svoistva prityagivayushchikh mnozhestv deformatsionnykh smeshchenii instrumenta v traektoriyakh formoobrazuyushchikh dvizhenii pri tochenii izdelii [The properties of attracting sets of tool deformation displacements in the trajectories of the shape-generating movements in turning]. *Izvestiya vysshikh uchebnykh zavedenii. Mashinostroenie = BMSTU Journal of Mechanical Engineering*, 2022, no. 3 (744), pp. 15–30. DOI: 10.18698/0536-1044-2022-3-15-30.

56. Zakovorotny V.L., Gvindjiliya V.E. Vliyanie dinamiki rezaniya na vybor tekhnologicheskikh rezhimov, obespechivayushchikh minimal'noe iznashivanie rezhushchikh instrumentov [Influence of cutting dynamic on the selection of the technological regimes to ensure minimal wear of cutting tools]. *Obrabotka metallov (tekhnologiya, oborudovanie, instrumenty) = Metal Working and Material Science*, 2020, vol. 22, no. 4, pp. 54–70. DOI: 10.17212/1994-6309-2020-22.4-54-70. (In Russian).

57. Biderman V.L. *Prikladnaya teoriya mekhanicheskikh kolebanii* [Applied theory of mechanical oscillations]. Moscow, Vysshaya shkola Publ., 1972. 416 p.

58. Makarov A.D. *Optimizatsiya protsessov rezaniya* [Optimization of cutting processes]. Moscow, Mashinostroenie Publ., 1976. 278p.

59. Zakovorotny V.L., Gvindjiliya V.E. Opredelenie optimal'nykh koordinat pereklyucheniya tsiklov obrabotki na metallorezhushchikh stankakh [Determination of optimal coordinates for switching processing cycles on metal-cutting machines]. *Obrabotka metallov (tekhnologiya, oborudovanie, instrumenty) = Metal Working and Material Science*, 2021, vol. 23, no. 1, pp. 56–67. DOI: 10.17212/1994-6309-2021-23.1-56-67.

Conflicts of Interest

The authors declare no conflict of interest.