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Theoretical study of the curvature of the treated surface during oblique milling with prefabricated milling cutters

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ABSTRACT

Introduction. The paper discusses the methods of processing large parts having curved convex surfaces with a rectilinear guide on multi-coordinate *CNC* machining centers using the touch method with a discrete motion of the tool feed along the profile of the part. It is shown that the main disadvantages of this method are lower productivity, which is due to the presence of discrete tool motions between cycles of its translation mode, where the value of discrete tool motion for a given processing accuracy depends on the curvature of the surface being processed. To improve processing performance, it is proposed to use prefabricated disc cutters equipped with replaceable polyhedral inserts (*RPI*) with rectilinear cutting edges. Its installation in the cutter body with non-zero angles of inclination of the main cutting edge, in combination with an additional rotation of the cutter, during processing, along the direction of the translational feed movement, allows you to obtain a concave surface and ensure a tighter fit of the producing surface of the tool and the machined surface of the part. **The aim** of the work is to reduce the error of approximation of the profile when it is processed using the touch method with discrete motion of prefabricated disc cutters along the profile and, consequently, to ensure workpiece the possibility of increasing the step of tool movement along the profile being formed to improve processing performance. **Research methods:** geometrical theory of designing metal-cutting tools. **Results and discussion.** The regularities established in the work made it possible to create a method for determining the angle of inclination of the main cutting edge of the *RPI* milling cutter and the angles of rotation of the milling cutter along the direction of translational feed movement during line-by-line processing of extended sections of parts with a curved profile on multi-coordinate *CNC* machines by turning the milling cutter to ensure the best fit of its producing surface to the surface being processed at the point of its contact, to reduce the approximation error processed profile and improve processing performance, due to the possibility of increasing the tool movement step.

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Introduction

Currently, machining of large pieces that have curved convex surfaces with a linear guide within the conditions of individual, small-series and repair production is performed with the help of multi-axis *CNC* machining centers due to the economic inexpediency of using special equipment. In this case, the formation of the parts’ surface can be performed using the touch method with a continuous feed movement of the tool along the profile of the part (fig. 1, *a*) or with a discrete movement of the tool (fig. 1, *b*).

For example, when milling parts with a thickness less than the height of the cutter, machining can be performed using the touch method with a continuous tool feed movement along the profile of the part (see fig. 1, *a*); and while milling parts with a larger thickness, the touch method with a discrete feed movement along the profile of the part can be applied (see fig. 1, *b*), where the milling cutter performs cyclic recipro-

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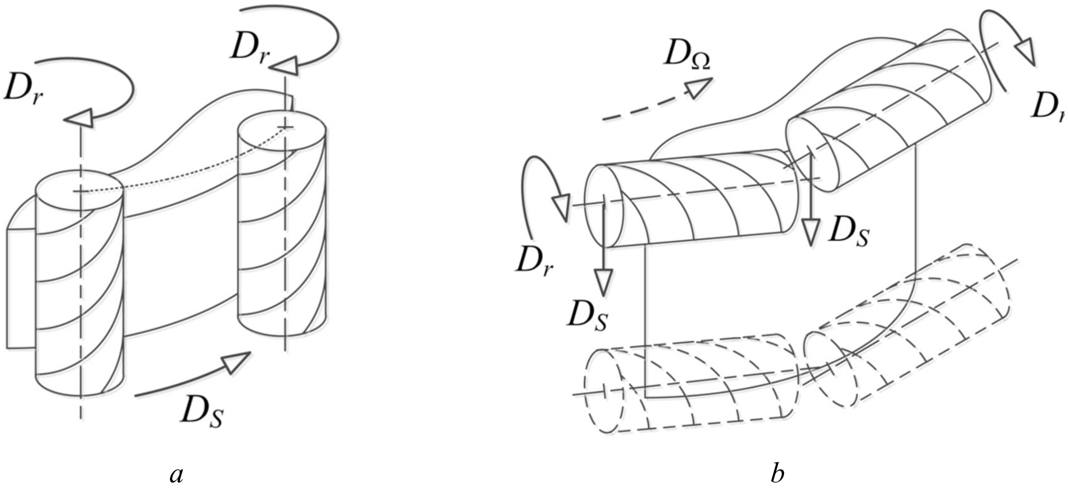


Fig. 1. The formation of the surface of the part by the touch method with the feed movement of the tool:

a – continuous; *b* – discrete

cal feed movements perpendicular to the profile of the part and shifts with each cycle along the part profile at a given discrete value depending on the required machining accuracy. An example of such parts is spur coarse pitch gears with pitch values greater than 9 mm and a face width greater than 50 mm, for which the machining according to the first method is effortful.

The machining of parts by the touch method with a continuous tool feed movement along the profile of the part has become widespread; there is a large number of works devoted to this problem [1–6]. The issues of machining by the touch method with discrete feed movement along the profile of the part due to the lower prevalence of shaped parts with heavy thickness are less studied [7–10]. The main disadvantage of this method is lower machining speed, which is due to the presence of discrete tool movement between the cycles of reciprocal movements. Moreover, the value of discrete tool movement Δ_Ω for a set machining accuracy depends on the value of the surface curvature being formed (fig. 2), which leads to a lower machining speed.

To increase the machining speed in this case, it is advisable to use milling cutters having a concave shape of the generating surface, which ensures its tighter fit to the surface to be machined. The generating surface is the one formed by the shape-generating cutting edge of the milling cutter due to its primary motion, i.e. the motion that determines the cutting speed [11].

However, with regard to the designs of prefabricated side milling or face milling cutters equipped with replaceable polyhedral inserts (*RPI*), it can be said that there are no *RPIs* of a standard design with a concave cutting edge. In [12–16], it has been found that when a milling cutter with a linear *RPI* cutting edge

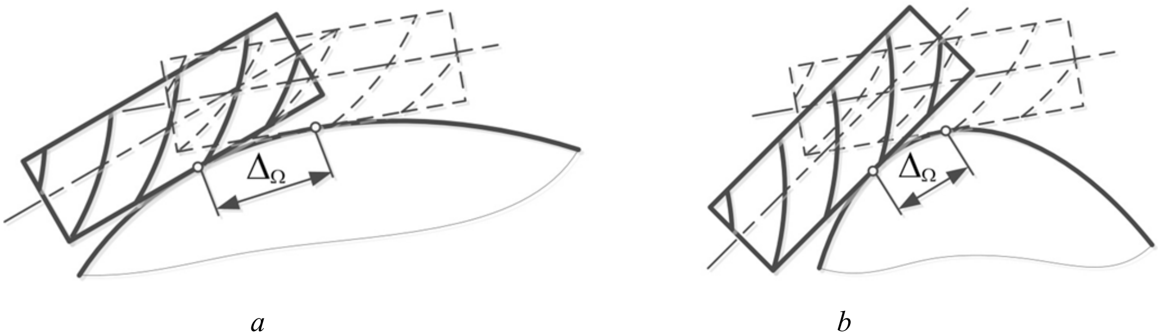


Fig. 2. The dependence of the value of the discrete tool movement on the curvature of the profile of the surface being processed:

a – at low curvature; *b* – at high curvature mounted in its housing is rotated at an angle of inclination of the main cutting edge that is different from zero ($\lambda \neq 0$), the generating surface takes a concave shape, and with an increase in the angle λ the curvature of this surface becomes larger. Moreover, the works [15, 16] show that the curvature of the generating surface of such cutters can be increased by turning the cutter along the direction of translation feed movement by the angle ξ, thereby implementing a scheme of oblique milling, for which it is assumed to use five-axis machining centers. Therefore, it is advisable to develop a milling cutter design by selecting the inclination angle of *RPI's* main cutting edge, in which the balance of the generating surface curvature and the least curvature of the convex surface profile being machined will be reached. By rotating the milling cutter for the calculated angle ξ while milling, this balance will also be reached along the entire profile. The implementation of this approach requires studying the influence of the cutter parameters (diameter, angle of inclination of the main cutting edge) and the angle of inclination of the cutter along the direction of the translational feed movement ξ on the change in the curvature of the machined surface (principal radii of curvature).

From the foregoing, the purpose of this study can be formulated as reducing the error in approximating the profile of the workpiece when it is machined by the touch method with discrete movement of interlocking side or end mills along the profile and, as a result, providing the possibility of increasing the tool approach increment along the formed profile to increase processing productivity.

The objective is to carry out a theoretical study of changes in the curvature of the part surface machined during oblique milling with interlocking side mills equipped with *RPIs*, as well as to develop a method for determining the inclination angle of *RPI* main cutting edge of a milling cutter and the rotation angles of a milling cutter along the direction of translation feed movement, ensuring the best fit between the producing surface of the cutter and the surface of the part in its points of contact.

Methodology

To carry out this study, a model of a prefabricated milling cutter with a nominal diameter (*d*), consisting of one *RPI* installed in the mill body with a taper lead angle (φ) and the side rake angle (λ) was developed (fig. 3).

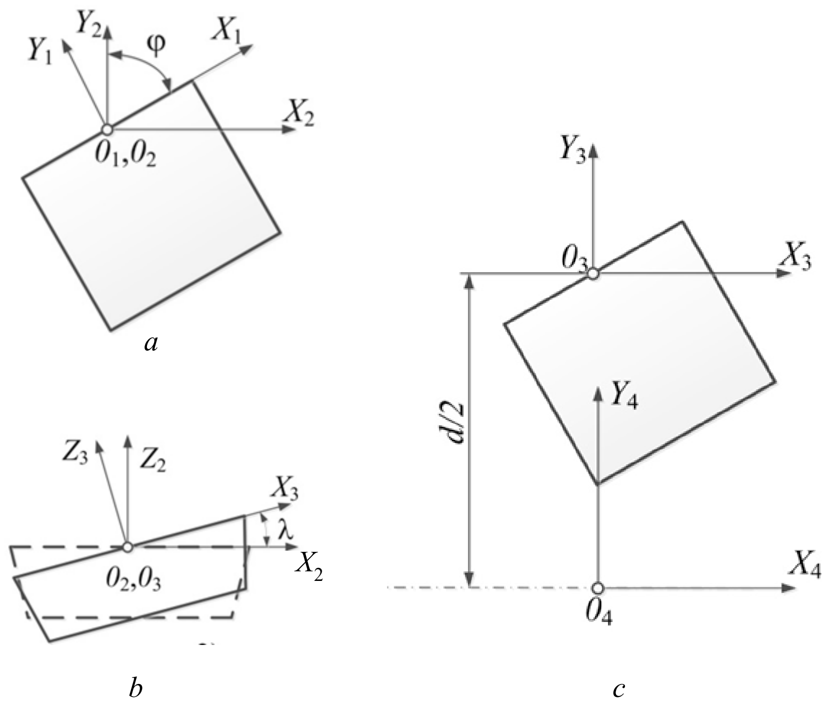


Fig. 3. Simulation scheme of sequential installation of the *RPI* in the milling cutter body:

a – setting the taper lead angle; *b* – setting the side rake angle; *c* – installation according to a given diameter in the milling cutter body

The equation of *RPI* cutting edge which determines the generating surface of the milling cuter under study is described in its own coordinate system ($X_1Y_1Z_1$), as

$$\bar{r}_1(t) = [t \quad 0 \quad 0 \quad 1]^{\mathrm{T}}, \tag{1}$$

where t is a length parameter of *RPI* cutting edge.

The equation of *RPI* cutting edge (1) is sequentially converted into the coordinate system of the mill body ($X_4Y_4Z_4$), taking into account the specified taper lead angle (fig. 3, position 1), the side rake angle (fig. 3, position 2) and the diameter of the cutter (fig. 3, position 3).

$$\bar{r}_4(t) = A_{43}^{\{2\}}(d/2)A_{32}^{\{5\}}(\lambda)A_{21}^{\{6\}}(\pi/2-\varphi)\cdot\bar{r}_1(t), \tag{2}$$

where $A_{43}^{\{2\}}(d/2)$ is the matrix that determines the installation of *RPI* on a specified diameter of the cutter d in the coordinate system of the mill body($X_4Y_4Z_4$):

$$A_{43}^{\{2\}}(d/2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$A_{32}^{\{5\}}(\lambda)$ is the matrix specifying the rotation of *RPI* relative to the axis OX_3 of the coordinate system ($X_3Y_3Z_3$) to provide a set side rake angle:

$$A_{32}^{\{5\}}(\lambda) = \begin{bmatrix} \cos\lambda & 0 & \sin\lambda & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\lambda & 0 & \cos\lambda & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$A_{21}^{\{6\}}(\pi/2-\varphi)$ is the matrix that determines the rotation of *RPI* relative to the axis OX_2 of the coordinate system $X_2Y_2Z_2$ to attain a set major cutting edge angle:

$$A_{21}^{\{6\}}(\pi/2-\varphi) = \begin{bmatrix} \cos(\pi/2-\varphi) & -\sin(\pi/2-\varphi) & 0 & 0 \\ \sin(\pi/2-\varphi) & \cos(\pi/2-\varphi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

By specifying the rotation of *RPI* cutting edge (2) relative to the tool axis, we obtain the equation of the generating surface of the milling cutter under consideration

$$\bar{r}_f(t,\theta) = A_{f5}^{\{5\}}(\pi/2)A_{54}^{\{4\}}(\theta)\bar{r}_4(t) \tag{3}$$

where θ is the angular parameter of the milling cutter generating surface;

$A_{f5}^{\{5\}}(\pi/2)$ is the matrix specifying the rotation of the coordinate system of the working tool to align axis Z_f with the axis of the milling cutter body:

$$A_{f5}^{\{5\}}(\pi/2) = \begin{bmatrix} \cos(\pi/2) & 0 & \sin(\pi/2) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\pi/2) & 0 & \cos(\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$A_{54}^{\{4\}}(\theta)$ is the matrix that specifies the rotation of *RPI* cutting edge profile $\bar{r}_4(t)$ by the angle θ.

Fig. 4 shows the result of developing the milling cutter generating surface with parameters $d = 30$ mm, $\varphi = 90°$ and $\lambda = 20°$ according to equation (3).

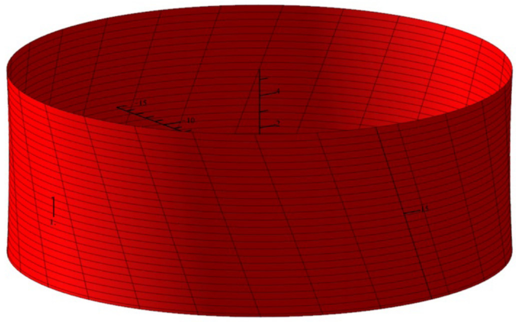


Fig. 4. Milling cutter generating surface

As a result of specifying the angle $\lambda \neq 0$, the generating surface of the milling cutter is a unipolar hyperboloid of rotation, characterized by the variability of the values of the principal radii of the surface curvature along the rotation axis, where its minimum value is reached at points $(0, \theta)$, for all $\theta \in [0; 2\pi]$.

The equation of the surface machined with the translation tool feed movement along axis X_0 of the part and a set value of the milling cutter rotation by the angle ξ (during oblique milling) is developed on the basis of the shaping equation

$$\bar{r}_0(x,t,\theta) = A^{\{1\}}(x)A^{\{5\}}(\xi)A^{\{6\}}(-\theta)\bar{r}_f(t,\theta) \tag{4}$$

where x is the parameter of the milling cutter travel along axis X_0;

$A^{\{1\}}(x)$ is the matrix that specifies the travel of the milling cutter along axis X_0:

$$A^{\{1\}}(x) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$A^{\{5\}}(\xi)$ is the matrix of the milling cutter rotation along the direction of translation feed movement by angle value ξ:

$$A^{\{5\}}(\xi) = \begin{bmatrix} \cos\xi & 0 & \sin\xi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\xi & 0 & \cos\xi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$A^{\{6\}}(-\theta)$ is the matrix that specifies the rotation of the milling cutter:

$$A^{\{6\}}(-\theta) = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) & 0 & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

To take into account the connection of the envelope of the form $\theta = \theta(x,t)$, based on equation (4) we compose and solve the equation with respect to parameter θ:

$$\begin{vmatrix} \bar{i}_0 \dfrac{\partial \bar{r}_0(x,t,\theta)}{\partial x} & \bar{j}_0 \dfrac{\partial \bar{r}_0(x,t,\theta)}{\partial x} & \bar{k}_0 \dfrac{\partial \bar{r}_0(x,t,\theta)}{\partial x} \\ \bar{i}_0 \dfrac{\partial \bar{r}_0(x,t,\theta)}{\partial t} & \bar{j}_0 \dfrac{\partial \bar{r}_0(x,t,\theta)}{\partial t} & \bar{k}_0 \dfrac{\partial \bar{r}_0(x,t,\theta)}{\partial t} \\ \bar{i}_0 \dfrac{\partial \bar{r}_0(x,t,\theta)}{\partial \theta} & \bar{j}_0 \dfrac{\partial \bar{r}_0(x,t,\theta)}{\partial \theta} & \bar{k}_0 \dfrac{\partial \bar{r}_0(x,t,\theta)}{\partial \theta} \end{vmatrix} = 0 \tag{5}$$

which makes it possible to represent equation (4) as a bivariate function:

$$\bar{r}_0(x,t) = A^{\{1\}}(x)A^{\{5\}}(\xi)A^{\{6\}}\left(-\theta(x,t)\right)\bar{r}_f\left(t,\theta(x,t)\right). \tag{6}$$

Fig. 5 shows the graphs of function $\theta(x, t)$ at $x = 0$ for the milling cutter with parameters $d = 30$ mm, $\varphi = 90°$, $\lambda = 20°$and the angle of the milling cutter rotation ψ being equal to $0°$ (line 1) and $20°$ (line 2). Fig. 6 shows the results of modeling the milling cutter generating surface when it is rotated by the angle of $\psi = 20°$ and the nominal machined surface.

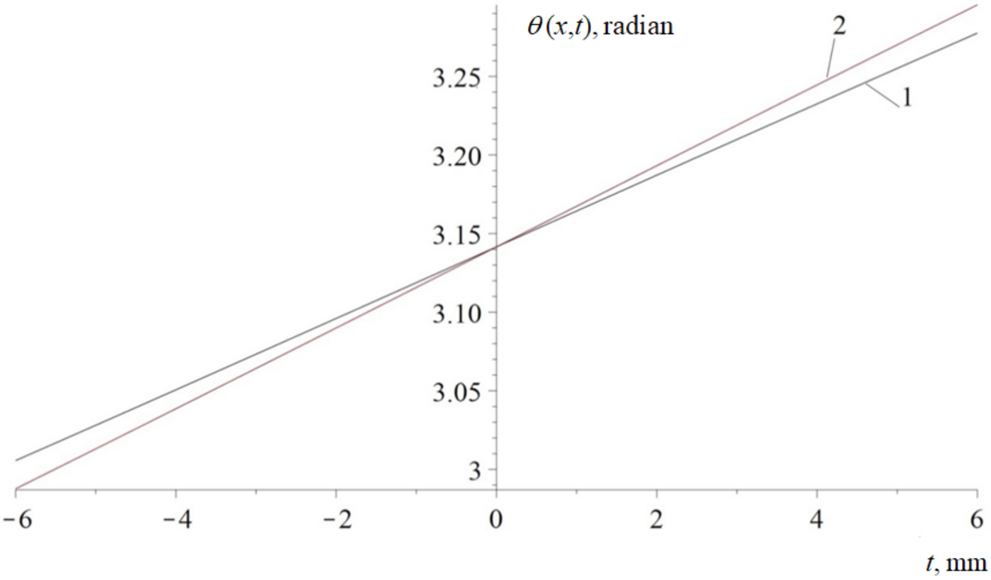


Fig. 5. Function graphs $\theta(x, t)$ at $x = 0$

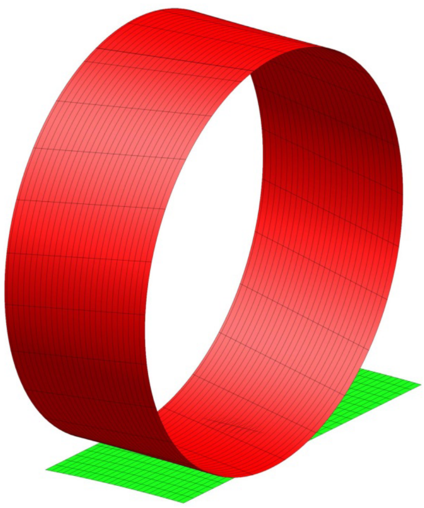


Fig. 6. Modeling of the milling cutter's producing surface and the nominal machined surface

We estimate the curvature of the surface machined by calculating its main curvatures (k_1 и k_2), which are the solution of the equation

$$k^2 - 2H \cdot k + K = 0, \tag{7}$$

where

$$H = \frac{LG - 2FM + EN}{2(EG - F^2)}, \tag{8}$$

$$K = \frac{LN - M^2}{EG - F^2}, \tag{9}$$

where E, F, G are the coefficients of the first quadratic form (g) of the machined surface (6), described by the equation:

$$g = E \cdot dx^2 + 2F \cdot dx \cdot dt + G \cdot dt^2, \tag{10}$$

where

$$E = \left(\frac{\partial \overline{r}_0(x,t)}{\partial x}\right)^2; \; F = \frac{\partial \overline{r}_0(x,t)}{\partial x}\frac{\partial \overline{r}_0(x,t)}{\partial t}; \; G = \left(\frac{\partial \overline{r}_0(x,t)}{\partial t}\right)^2, \tag{11}$$

L, M, N are the coefficients of the second quadratic form of the machined surface (6), described by the equation:

$$q = L\,dx^2 + 2M\,dx\,dt + N\,dt^2, \tag{12}$$

where

$$L = \frac{\partial \overline{r}_0(x,t)}{\partial x}\frac{\overline{n}(x,t)}{\sqrt{EG - F^2}}; \tag{13}$$

$$M = \frac{\partial \overline{r}_0(x,t)}{\partial x \partial t}\frac{\overline{n}(x,t)}{\sqrt{EG - F^2}}; \tag{14}$$

$$N = \frac{\partial \overline{r}_0(x,t)}{\partial t} \frac{\overline{n}(x,t)}{\sqrt{EG - F^2}}, \tag{15}$$

where $\overline{n}(x,t)$ is the normal to the machined surface;

$$\overline{n}(x,t) = \frac{\dfrac{\partial \overline{r}_0(x,t)}{\partial x} \dfrac{\partial \overline{r}_0(x,t)}{\partial t}}{\left| \dfrac{\partial \overline{r}_0(x,t)}{\partial x} \dfrac{\partial \overline{r}_0(x,t)}{\partial t} \right|}. \tag{16}$$

For the convenience of perception, in the future, instead of (k_1 и k_2), we will consider the principal radii of the machined surface curvature $R_1 = k_1^{-1}$ and $R_2 = k_2^{-2}$.

Studying the principal radius of the machined surface curvature in cross section (fig. 7) for the milling cutter with d = 30 mm, φ = 90°, λ = 20° and ξ = 0° confirmed that it reaches the lowest value at the point of the surface being formed by the middle of the *RPI* cutting edge (t = 0) and increases as it moves away from the middle (curve 1).

The study also showed (fig. 7) that an increase in the angle of rotation of the cutter (at ξ = 20°, curve 2) leads to a decrease in the principal radius of curvature. Fig. 8 shows graphs of the change in the main curvature of the machined surface in the cross section (at the point t = 0) at different angles of rotation of the cutter with parameters ξ ∈ [0;45°] with parameters d = 30 mm, φ = 90° and angle λ = 10° (line 1) and λ = 20° (line 2).

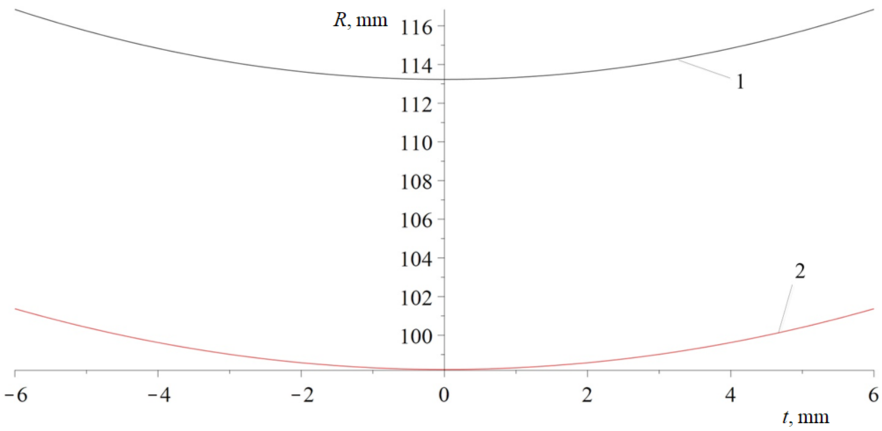


Fig. 7. Change of the main radius of curvature (R) on different sections of the processed surface in cross-section

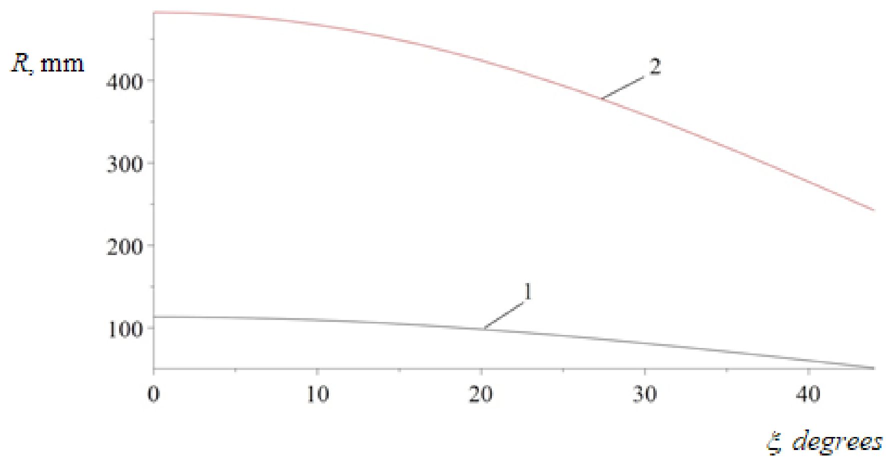


Fig. 8. Change of the main radius of curvature (R) of the processed surface in cross-section from the angle ξ

The use of the obtained equations makes it possible to calculate the inclination angle of *RPI* side rake angle and the rotation angles of the milling cutter along the direction of the translational feed movement that will provide the best fit of the generating surface of the milling cutter and the part surface at its contact points. It is advisable to perform this calculation in the following sequence:

1) for a specified diameter of the milling cutter, using equation (7), we calculate the minimum value of the angle λ (at $\xi = 0°$) that provides the best fit of the generating surface of the milling cutter at the point of the surface with the least curvature (the largest principal radius of curvature) according to the condition:

$$R_{d\,\max} \approx R(\lambda, \xi), \tag{17}$$

where $R_{d\,\max}$ is the largest radius of the profile curvature of the surface being formed; $R(\lambda, \xi)$ is the principal radius of the machined surface curvature by the milling cutter at a specified angle λ and the angle of the cutter rotation ξ.

2) at the specified value of angle λ, we calculate the inclination angles of the milling cutter ξ at the remaining points of the profile of the surface being formed according to condition (17). In case when the surfaces to be machined have a large value range of the principal radius of curvature, it may not be possible to achieve strict equality (17) at all points. Then for these points it is necessary to take angle ξ equal to the largest possible value ($\xi = 45°$).

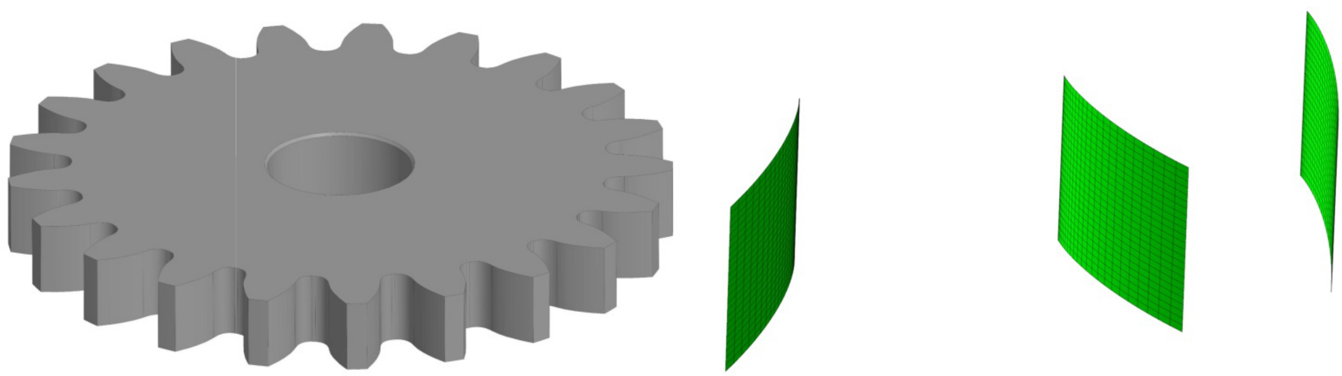
Results and discussion

The practical application of the constructed models and established regularities will be considered through the example of machining the involute surface of a spur coarse pitch gear (fig. 9, *a*) with pitch of 9 mm, 21 teeth and a face width equal to 50 mm and with the following equation:

$$\overline{r}_d(u, v) = \left[R_0(\cos u + u \sin u) \quad R_0(\sin u - u \cos u) \quad v \quad 1\right]^{\mathrm{T}}, \tag{18}$$

where R_0 is the radius of the generating circle of the gear wheel; for our wheel R_0 = 197.3 mm.

The calculation of the principal radius of curvature of this surface in the across-track direction has shown that its size at $u \in [0; 0{,}61]$ varies from 0 to 120.5 mm. Taking the diameter of the milling cutter as 30 mm, the calculation of the minimum angle value λ (at $\xi = 0°$) is performed to attain the best fit of the generating surface of the milling cutter to the surface point with the least curvature (the largest principal radius of curvature) u = 0.61 according to the condition (17). By specifying the incremental step of angle λ equal to 30' and the angle $\xi = 0°$, it is found that condition (17) is fulfilled at $\lambda = 19°$ with $R(19°)$ = 126.5 mm. Further, with the set the angle $\lambda = 19°$, the calculation of inclination angles of the milling cutter ξ is performed for the profile points of the surface formed (fig. 10).



a *b*

Fig. 9. Spur gear:
a – a geometric model; *b* – the result of modeling the surface of the teeth according to (17)

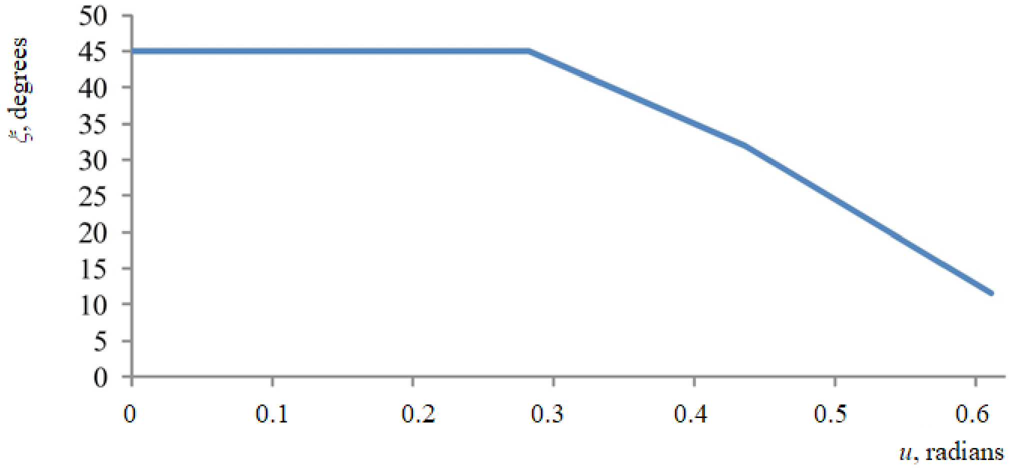


Fig. 10. Change of angle ξ at ξ = 19° and $u \in [0; 0.61]$

Fig. 11 demonstrates the position of the milling cutter during the formation of different sections of the wheel tooth surface being machined in accordance with the calculated rotation angles of the milling cutter (fig. 10).

In fig. 11, position 1 corresponds to the point of the tooth surface profile $u = 0$ rad, position 2 corresponds to the point of the tooth surface profile $u = 0.44$ rad, position 3 corresponds to the point of the tooth surface profile $u = 0.61$ rad. It follows from fig. 11 that with increase in the curvature of the surface being machined, the rotation angle of the milling cutter becomes larger.

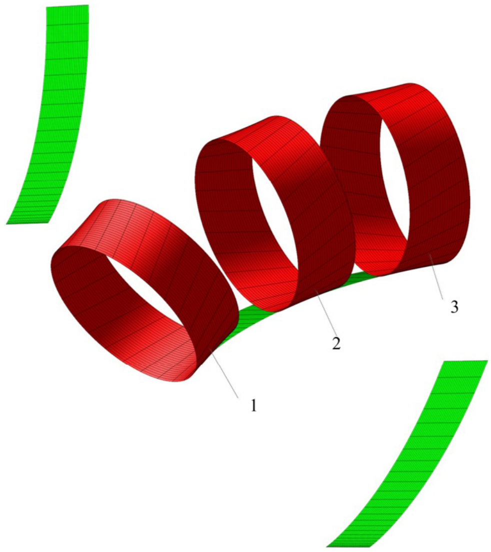


Fig. 11. Installation of the milling cutter across points of the surface being formed

Conclusion

The established regularities of changing the principal radius of curvature of the surface machined in cross-section in case of the line-by-line machining of extended sections of parts with a curved profile (in particular, convex surface sections of the parts) on multi-axis *CNC* machines by rotating the milling cutter to ensure the best fit of its generating surface to the machined surface at its point of contact. It also ensures a decrease in an approximation error of the surface profile of the machined surface and improves the processing productivity due to the possibility of increasing the tool approach increment along the formed profile.

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Conflicts of Interest

The authors declare no conflict of interest.