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Modeling the interrelation of the cutting force with the cutting depth and the volumes of the metal being removed by single grains in flat grinding

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ABSTRACT

Introduction. The model for calculating the cutting force is the basis of the modules of *CAM*-systems related both to the predicting processing errors on metal-cutting machines for specified grinding conditions, and to the optimizing all parameters of the technological mode (parameters of cutting modes, cutting tools, etc.). However, due to the lack of an adequate model for calculating the cutting force presented in engineering form, such modules have not yet been developed not only for flat grinding operations, but also for all other types of metalworking. It is challenging to obtain an adequate cutting force model for flat grinding operations because it is necessary to establish the interrelation between the machine parameters of the macrocutting modes (feed, cutting speed) of the grinding wheel with the parameters of the microcutting modes — the sets of cutting grains of the wheel associated with the plastic deformation of the metal in the shear zone, microvolumes of the metal being removed and the geometry of the cutting part of the abrasive grains. **The purpose** of this work is to develop a force model establishing the interrelation of the cutting force with the cutting depth and the volumes of the metal being removed by single grains and the wheel as a whole on the basis of the integration of microvolumes and microforces when metal being removed by the wheel grains. **Research methods.** *The subject of the research* is the mathematical modeling of the interrelation between the cutting force and cutting modes with the parameters of microcutting by a group of single grains, based on the equality of work when metal being removed of the same volume. *The methodological basis of the research* is the connection between the work (energy) spent on the plastic deformation of metal by a single grain, the intensity of stresses, the intensity of deformation rates and the volume of the metal being removed by the wheel as a whole, established by *S.N. Korchak*. **Results and discussion.** *The result of the study* is an analytical model that reliably and adequately establishes the interrelation between the cutting force and the cutting depth, cutting modes, wheel characteristics, physical and mechanical properties of the processing material, and other main technological parameters. *The field of application of the results* is the possibility of using the cutting force calculation model, presented in this paper, as a basis for the development of a module for a *CAM*-system (a digital twin of the machining), which would allow to perform the calculation and design of optimal technological parameters of the flat grinding operation, as well as cutting modes testing according to the criterion of processing accuracy of a parts batch, considering the influence of various variable factors and real processing conditions within the manufacturing process.

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Introduction

Flat grinding is a finishing operation subjected to high requirements on the accuracy and quality of machining. These requirements are difficult to satisfy in flat grinding because of the need to control three feeds (fig. 1); the presence of grooves, holes, etc. on machined surfaces; simultaneous machining of several workpieces during the operation; and other factors. The accuracy and quality of the machined surface directly depend on cutting modes. Currently, cutting modes are assigned using a *CAM* system, which is

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based on digitized reference literature. It is currently impossible to test cutting modes for compliance with the requirements of process design and production charts using *CAM* systems in a virtual environment. Manufacturers are thus forced to carry out trial processing of workpieces according to the cutting modes set by the process engineer. At the same time, cutting modes are often diminished to a safe level in order to sequentially meet the drawing requirements in the manufacture of a batch of parts, which undoubtedly leads to lost productivity. Experimental verification increases the duration of the preparatory production stage, additional time and material costs.

This problem has not yet been solved due to a lack of a wide-range analytical engineering model that reliably establishes the mathematical relationship between cutting force and cutting depth, the volume of metal being removed, cutting modes, the characteristics and geometric parameters of the grinding wheel, the physical and mechanical properties of the material being machined, etc.

S.N. Korchak [1] established the relationship between the cutting force of a single grain of an abrasive wheel and the physical and mechanical properties of the material being machined through shear and compression stresses, taking into account the temperature in the cutting zone. Similarly, *Yu. M. Zubarev et. al.* [2] and *Filimonov* [3] obtained calculation dependencies to find the cutting force for grinding operations, taking into account friction (internal friction coefficient, friction angle, etc.). The dependences obtained in [1] were used to develop models to calculate the cutting force for flat grinding [4–5]. Several authors have presented force dependences that take into account the processes of blunting and wear of the abrasive grains in models of the cutting force occurring during grinding [6–14]. Others have considered the impact of dynamic loads on the cutting force arising from the "contact stiffness" of manufacturing, the non-stationary nature of abrasive machining, the variable stiffness of manufacturing, etc. [15–23]. In studies by *Leonesioa et. al.* [15] and *Li* and *Shin* [16], the cutting force is calculated with consideration to the rigidity of the technological system and dynamic loads. *Patnaik et. al.* [17] calculated the cutting force by taking into account the friction coefficient of flat grinding. The papers do not present engineering formulas, which complicates its practical application in mechanical engineering.

We can highlight a number of papers on the development of a cutting force model for flat grinding operations. *Voronov* and *Veidun* [24] proposed a mathematical model of flat grinding with a disk with abrasive grains distributed over a cylindrical surface with random geometric characteristics. *Nosenko et. al.* [25] presented a dynamic mathematical cutting force model taking into account the wear of the working wheel surface during flat grinding. *Danilenko* [26] studied the components of the cutting force that occur during flat grinding for a narrow range of materials (mainly various titanium alloys). *Li* and *Yang* [27] experimentally assessed changes in cutting force and roughness for flat grinding of several grades of steel. *Liu et. al.* [28] experimentally assessed the impact of cutting speed on cutting force and wear of a grinding wheel.

An analysis of the literature showed that, despite the abundance of analytical models linking cutting forces to the depth of cutting by single cutting grains, there are still no adequate engineering models for calculating the cutting force for a given cutting depth during grinding for the wheel as a whole. The proposed models calculate a certain cutting depth and force when metal is cut with a single grain of an abrasive wheel, depending on the number of grains and other factors, in the absence of reliable a priori information on the number of cutting grains and the volumes of metal removed. These formulas cannot be used to calculate the cutting depth of a grinding wheel and the resulting cutting force or the allowance to be removed during the operation in several passes.

Program depth feed changes tenfold in a stepped cycle of flat grinding (for example, from 0.05 mm/stroke at the first stage of the cycle to 0.001 mm/min at the last stage). The number of cutting grains and the cutting depth should also be significantly reduced. However, this does not affect the cutting depth by the wheel grains in the considered force models. Considering that there are an excessive number of cutting grains on the working surface of the wheel [29], some grains pass through the contact zone without metal removing. With a tenfold decrease in the depth feed, the number of excess grains increases. There are no available methods for calculating the number of excess grains and the cutting depth with a tenfold decrease in the depth feed. In addition, it is not taken into account that the volume of the metal layer removed from the workpiece has a finite value. Models also do not calculate the volume of metal removed during microcutting; the removed metal is not summed and it is unclear when the process of removing a given metal volume will end. A significant omission in the discussed models is that the cutting force does not change depending on the wheel hardness.

Thus, there is no scientific approach establishing the relationship of the cutting forces and the depths of the metal cut by a single grain with the stock removal and the cutting force that occurs during grinding with a wheel as a whole. As a result, there is no analytical engineering model, which calculates the relationship between cutting force and cutting depth in flat grinding operations.

It is challenging to obtain an adequate cutting force model for flat grinding operations because it is necessary to establish the interrelation between the machine parameters of the macrocutting modes (feed, cutting speed), adjusted on the machine control panel, and parameters of the microabrasive grain cutting conditions, associated with plastic metal deformation in the shear zone, the physical and mechanical properties of the metal being machined, the back rake angle and front clearance of the grain cutting edge, blunting of the cutting edge along the back edge grain surface, cutting speed, the parameters of the contact zone of the cutting tool with the workpiece, etc.

For flat grinding in particular, the volume of metal removed from the workpiece and the parameters of the three machine feeds (depth, transverse, longitudinal) should be linked to microcutting parameters: the metal shear angle in the plastic deformation zone; the length and area of contact between the wheel and the workpiece; the variable depth of the metal cut by single grains of the wheel; the stochastic nature of metal being removed by an excess number of wheel grains; the geometry of the cutting grains of the grain cutting edge, the blunting area on the back edge surface; the strength of the metal being machined; as well as the total microvolume of metal removed by all grains.

The purpose of this paper is to establish the relationship between the cutting force, the cutting depth, and the volume of metal removed by single grains in flat grinding. Let us consider the main modeling stages for calculating the cutting force in flat grinding based on the equality of the volume of metal removed by a set of single grains and the same volume of metal removed by the grinding wheel as a whole (the equality of the volumes of metal removed).

Research methods

We will use the model of the cutting force of a single abrasive grain as the foundation for modeling cutting force during flat grinding. Metal is removed by a single grain at energy costs, most often expressed in the form of grinding energy or power. The relationship between energy and power during plastic metal deformation, as established by *Korchak* are expressed as [1]:

$$A = \iiint_{\omega} \sigma \varepsilon d\omega, \qquad (1)$$

$$N = \frac{dA}{dt} = \iiint_{\omega} \sigma \dot{\varepsilon} d\omega, \qquad (2)$$

where A is the energy expended on the deformation of the metal volume ω, N·m (J); N is the power required to deform the metal ω, N·m/s (W); σ is the stress intensity in the moving volume of the deformed metal, N/m^2; $\dot{\varepsilon}$ is the deformation rate intensity, s^{-1}; ε is the deformation degree intensity; ω is the volume of deformed metal, m^3; t is the deformation time of the metal volume ω, s.

The assumptions made by *Korchak* are not bound to a specific type of grinding; it is also valid for the developed model of the cutting force occurring during flat grinding [1]. Fig. 1 shows the calculation scheme of a metal cutoff by the cutting edge of the abrasive grain of a wheel with a blunting area l_j. *Korchak* adopted a free cutting pattern due to the minor influence of edge effects along the length of the cutting edge, which exceeds a hundredfold the depth of cut by a single grain of a wheel [1]. It is assumed that the shear zone is a parallelogram, since the temperature and speed grinding parameters (cutting speed of 30...60 m/s and temperature of 600...1,500 ºC) transform the metal into a state similar to an ideal plastic, and the shear zone is narrowed into a thin strip, rather than a wedge as is seen in other types of cutting. According to *Korchak*, the thickness m of the shear zone is 1…5 µm [1]. As a result, it is also assumed for free cutting conditions that in the parallelogram shear zone (fig. 1), the stress intensity σ and deformation rate intensity $\dot{\varepsilon}$ are constant on average. The metal is sheared in the cutting zone if the abrasive grain is absolutely sharp, when the length of the cutting edge is equal to b_j and the length of the blunting area along the rear face is equal to zero, i.e. $l_j = 0$.

The area of the shear zone can be described by the thickness m of the zone itself, the cut depth a_s, and the shear angle β_1 (fig. 1). The energy of an absolutely sharp abrasive grain is expended on the plastic deformation of the metal of a volume ω_j in the shear zone during the shear time Δt, when the peak of the sharp grain of the wheel passes the distance h_j (see fig. 1, the distance between points O and O_1) at a speed of V_k. The volume of chips being removed (fig. 1 and 2, *a*) consists of a set of volumes ω_j.

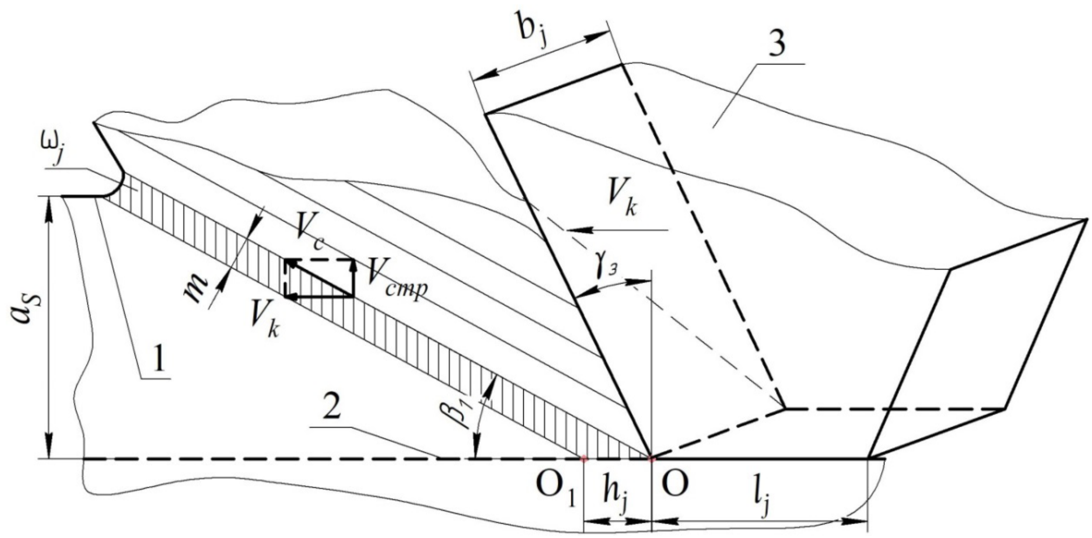


Fig. 1. Calculation model of plastic deformation in the shear zone when cutting with a single grain:

1 – outer metal surface; *2* – cutting grain motion pattern; *3* – cutting grain

We transform expression (1) with regard to the shift of the elementary metal volume ω_j in the cutting zone made by the j-th grain of the wheel to calculate the energy A_j spent on the plastic deformation of the metal volume ω_j (fig. 1):

$$A_j = \sigma\varepsilon\iiint\limits_{\omega} d\omega = \sigma\varepsilon\omega. \tag{3}$$

Let us express the power N_j of metal deformation in the shear zone through the increment of the elementary metal volume ω_j and the elementary shear time Δt:

$$N_j = \frac{dA}{dt} = \sigma\dot{\varepsilon}\frac{d\omega}{dt} = \sigma\dot{\varepsilon}\omega_j, \tag{4}$$

given that

$$\omega_j = \frac{a_s m b_j}{\sin\beta_1}, \tag{5}$$

where a_s is the thickness of the cutoff made with a sharp single grain (fig. 1 and 2, b), m; m is the thickness of the metal shear zone in the cutting zone, m; β_1 is the slope angle of the reference shear plane, deg. (fig. 2, *b*); b_j is the length of the cutting edge of a single abrasive grain, m; N_j is the power of deformation of the metal volume ω_j in the shear zone, N·m/s (W).

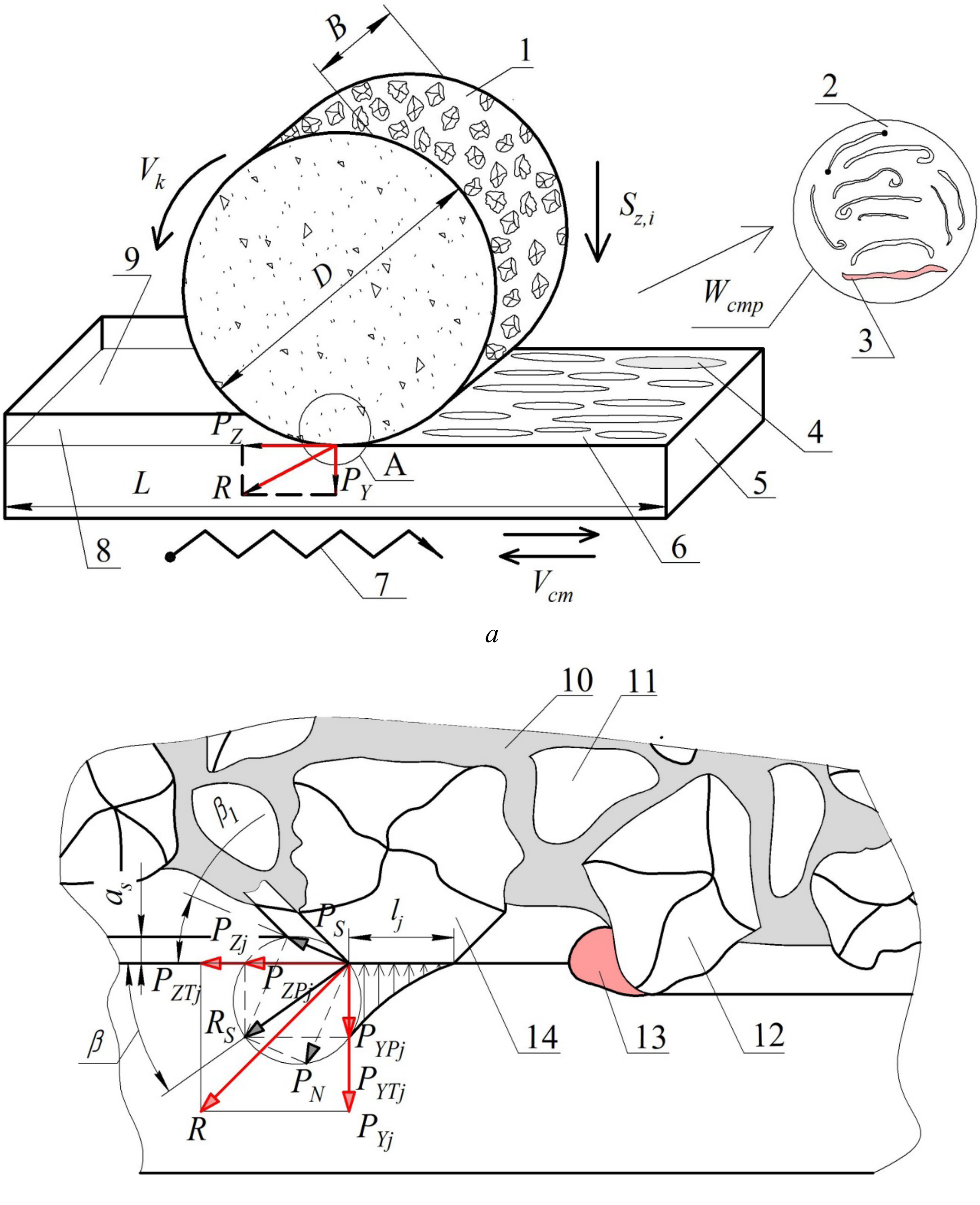


Fig. 2. Flat grinding (*a*), a cutoff with a single grain, diagram of stresses and force components (*b*) – view *A* in fig. 1, *a*:

1 – grinding wheel; *2* — total volume of chips being removed (shown conditionally); *3* – chips being removed by a single grain; *4* – grain trace; *5* – workpiece; *6* – machined surface; *7* – magnetic table; *8* – stock; *9* – surface being machined; *10* – bond; *11* – pore; *12* – real abrasive grain; *13* – chips cut by a single grain; *14* – ideal abrasive grain

Then, formula (4) will be written as:

$$N_j = \sigma\dot{\varepsilon}\frac{a_s m b_j}{\sin\beta_1}. \tag{6}$$

The plastic deformation of the metal volume ω_j in the shear zone results from the action of the tangential component of the cutting force P_{ZSj} of an absolutely sharp grain. Therefore, deformation power N_j is equal to the power of the force P_{ZSj}. We will use the scheme of a cutoff made by a conditionally sharp grain (Fig. 2, *b*) to express the power N_j:

$$N_j = R_{Sj} V_k \cos \beta = P_{ZSj} V_k, \tag{7}$$

where R_{Sj}, P_{ZPj} are the resultant and tangential components of the cutting force of the sharp grain, respectively, N; V_k is the wheel speed, m/s; β is the angle between R_{Sj} and vector V_k (fig. 2, *b*), degrees.

Let us find the tangential component of the force of cutting with a single grain by equating formulas (6) and (7).

$$P_{ZPj} = \sigma \dot{\varepsilon} \frac{a_s m b_j}{V_k \sin \beta_1}. \tag{8}$$

The radial component of the cutting force P_{YPj} with a single grain can be found by the formula:

$$P_{YPj} = R_{Sj} \sin \beta = P_{ZPj} \operatorname{tg}\beta, \tag{9}$$

$$P_{YPj} = \sigma \dot{\varepsilon} \frac{a_s m b_j}{V_k \sin \beta_1} \operatorname{tg}\beta. \tag{10}$$

During grinding, the abrasive grains of the wheel blunt, forming an area l_j on the back edge. The blunting area prevents the penetration of abrasive grains into the metal during cutting. An additional radial force P_{YTj} appears, which is necessary to deepen the blunt grain (fig. 2, *b*), without which metal shear is impossible. During cutting, friction occurs between the grain blunting area along the metal surface, which results in an additional tangential force P_{ZTj}. Therefore, it becomes necessary to take into account the influence of the blunting area on the total force, the radial cutting force P_{Yj}, and the tangential force P_{Zj}.

Let us use the model of the cutting force acting on an abrasive grain with a blunting area as the basis (figs. 1 and 2, *a*). Fig. 2, *a* shows a kinematic diagram of flat grinding. Many of the elements in fig. 2 are shown conditionally and enlarged in size for clarity (guide marks, abrasive grains, etc.).

Let us construct equations to find the components of the force of cutting by the wheel grains (fig. 2, *b* [1]):

$$P_{Yj} = P_{YPj} + P_{YTj}, \tag{11}$$

$$P_{Zj} = P_{ZPj} + P_{ZTj}. \tag{12}$$

Let us take into account the influence of the blunting area of the abrasive grain on the components of the cutting forces resulting from the friction force [1]:

$$P_{YTj} = \frac{\sigma}{3} b_j l_j, \tag{13}$$

$$P_{ZTj} = \mu P_{Ytj} = \mu \frac{\sigma}{3} b_j l_j, \tag{14}$$

where μ is the coefficient of the abrasive grain friction along the material being machined; l_j is the length of the grain blunting area, m.

We substitute formulas (9), (10), (13), and (14) into equations (11)–(12) to obtain the following equations:

$$P_{Yj} = \sigma \dot{\varepsilon} \frac{a_s m b_j}{V_k \sin \beta_1} \operatorname{tg}\beta + \frac{\sigma}{3} b_j l_j, \tag{15}$$

$$P_{Zj} = \sigma \dot{\varepsilon} \frac{a_s m b_j}{V_k \sin \beta_1} + \mu \frac{\sigma}{3} b_j l_j. \tag{16}$$

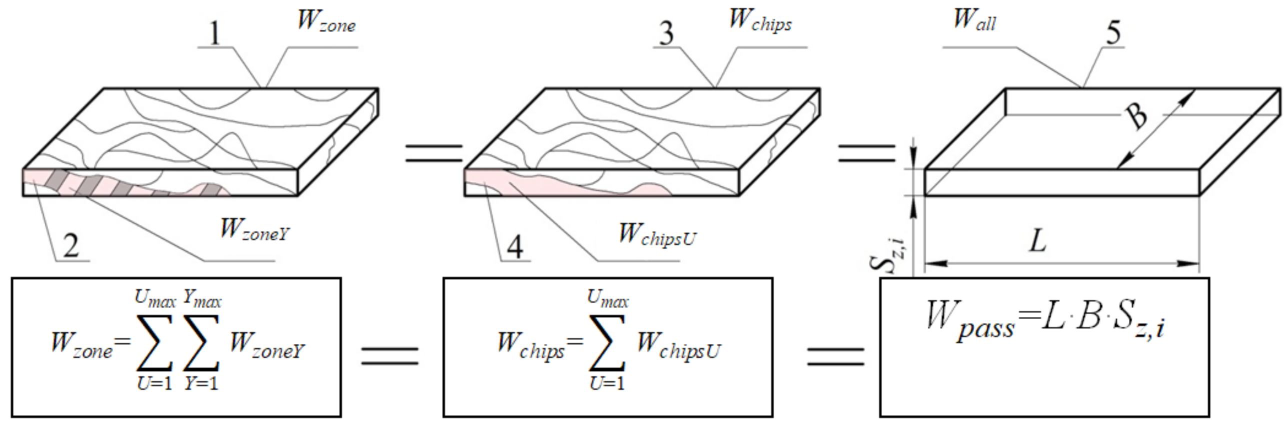
Formulas (15) and (16) are models for calculating the radial and tangential components of the cutting force during plastic deformation of the metal volume in the shear zone by a single blunt grain.

In models (15) and (16), the components of the cutting force are expressed not through the parameters of machine modes of cutting by the wheel as a whole (depth feed, axial, longitudinal), but through the parameters of microcutting in the shear zone of the metal and friction of the blunting area of a single grain. The first term in models (15) and (16) determines the cutting force, when the metal is cut with an absolutely sharp grain necessary for plastic deformation of the metal in the shear zone. The addend determines the cutting force needed to overcome the indentation and friction of the blunting area.

In flat grinding, metal is simultaneously removed by several grains (a group of single grains) in the contact zone between the wheel and the workpiece, so let us transform the microcutting parameters of many grains into the operating parameters of the wheel as a whole, based on the condition for ensuring the equality of energy expended to remove the same volume of metal by the grains and the wheel as a whole. In other words, let us assume that the volume of metal $W_{all\ (allowance)}$ removed during one pass of the table from the workpiece by a set of single abrasive grains is equal to the same volume of metal W_{all} removed by the grinding wheel as a whole (fig. 3). To perform such a transformation, let us make the following assumptions:

1) the volume of metal W_{all} being removed from the workpiece is equal to the sum of all volumes W_{zone} of the metal in the shear zone, i.e. $W_{all} = W_{zone}$ (fig. 3).

2) since the volume of the chips being cut consists of many metal shear layers in the cutting zone, the total volume of metal chips is $W_{chips} = W_{zone}$ and is also equal to $W_{all} = W_{chips} = W_{zone}$ (fig. 3).



* $U_{\max}$ ($Y_{\max}$) is the maximum possible number of chips (shear zones)

Fig. 3. The equality of volumes of metal being removed in shear zones, in the chips, and in the workpiece:
1 – the total volume of metal in the shear zones; *2* – the volume of metal in the shear zone; *3* – the total volume of chips being removed; *4* – the volume of chips cut by a single grain; *5* – the total amount of metal being removed during one pass of the table

3) the metal volume W_{all} is determined by the length of the workpiece L (mm), the width of the wheel working surface B (mm), and the depth feed $S_{z,i}$ (mm/pass), i.e. (fig. 3)

$$W_{all} = S_{z,i} LB = W_{chips} = W_{zone}. \tag{17}$$

4) the sum of cutting forces from all cutting grains that are in contact with the workpiece is equal to the cutting force for the wheel as a whole, i.e.:

$$P_Z = P_{\Sigma ZP} + P_{\Sigma ZT} = \sum_{j=1}^{J} P_{ZPj} + \sum_{j=1}^{J} P_{ZTj}, \tag{18}$$

$$P_Y = P_{\Sigma YP} + P_{\Sigma YT} = \sum_{j=1}^{J} P_{YPj} + \sum_{j=1}^{J} P_{YTj}, \tag{19}$$

where P_Y, P_Z are the radial and tangential components of the cutting force during grinding, respectively; $P_{\sum YP}$, $P_{\sum ZP}$ are the radial and tangential components of the cutting force during grinding when all grains are absolutely sharp, respectively; $P_{\sum YT}$, $P_{\sum ZT}$ are the radial and tangential components of the cutting force from the blunting areas during grinding, respectively.

5) the equality of energy spent on the removal of the same metal volume W_{all} by a set of single abrasive grains and by the grinding wheel as a whole.

Let us establish the relationship of energy on removing the metal volume W_{all}, separately for a group of grains and the wheel as a whole. According to the structure of model (12) of the cutting force P_{Zj} for a single grain, the total energy A_j (H·m) expended by the cutting force P_Z consists of the energy A_{Pj} expended by the force P_{ZPj} and the energy A_{Tj} expended by the force P_{ZTj}, i.e.:

$$A_j = A_{Pj} + A_{Tj}. \tag{20}$$

Similarly, the total energy $A_{\sum}$ from the tangential component of the cutting force for the wheel consists of the energy $A_{\sum P}$ expended in total by the forces P_{ZPj} from all the cutting grains in the contact zone and the energy $A_{\sum T}$ expended in total by the forces P_{ZTj} from all the cutting grains in the contact zone, i.e.:

$$A_{\Sigma} = A_{\Sigma P} + A_{\Sigma T}. \tag{21}$$

Then, the sums of the grinding energy from the cutting forces of single grains are equal to the grinding energy of the wheel from the total cutting force. Let us find the grinding energy expended on plastic metal deformation in the shear zone by the force P_{ZPj} at a distance L of the length of the machined surface (fig. 2, *a*):

$$A_{Pj} = P_{ZPj} L = \sigma\dot{\varepsilon}\frac{a_s m b_j}{V_k \sin\beta_1} L = \sigma\dot{\varepsilon}\frac{\omega_j}{V_k} L, \tag{22}$$

$$A_{\Sigma P} = \sum_{j=1}^{J} A_{Pj} = L\sum_{j=1}^{J} P_{ZPj} = \frac{\sigma\dot{\varepsilon}L}{V_k}\sum_{j=1}^{J}\frac{a_s m b_j}{\sin\beta_1} = \frac{\sigma\dot{\varepsilon}L}{V_k}\sum_{j=1}^{J}\omega_J = \frac{\sigma\dot{\varepsilon}LW_{np}}{V_k}, \tag{23}$$

where L is the length of the machined surface, m; ω_j is the volume of metal removed during cutting with the j-th single grain, m^3.

To establish the relationship between the grinding energy $A_{\sum P}$ expended by absolutely sharp grains to remove metal, we should find a common energy parameter inherent in single grains and the wheel as a whole. Metal removal rate Q, m^3/s is an appropriate parameter for our purposes. When cutting with the j-th single grain, the metal removal rate Q_j is equal to the ratio of the metal volume ω_j in the shear zone to its deformation time during shear time Δt, which is equal to a sharp grain moving across a distance h_j at a speed of V_k:

$$Q_j = \frac{\omega_j}{\Delta t}, \tag{24}$$

$$\text{at } \dot{\varepsilon} = \frac{\varepsilon}{\Delta t}, \tag{25}$$

where Q_j is the metal removal rate when cutting with the j-th single grain, m^3/s; Δt is the deformation time of the metal volume ω_j during the shear time, s.

Let us take into account formulas (24) and (25) in the equation:

$$A_{\Sigma P} = \frac{\sigma\dot{\varepsilon}L}{V_k}\sum_{j=1}^{J}\omega_J = \frac{\sigma\varepsilon L}{V_k}\sum_{j=1}^{J} Q_J, \tag{26}$$

The sum of the metal removal rates by a group of grains in the contact zone is equal to the metal removal rate of the wheel Q_w:

$$Q_w = \sum_{j=1}^{J} Q_j , \qquad (27)$$

where Q_w is the metal removal rate by the wheel, m^3/s.

Then formula (4) will be as follows:

$$A_{\Sigma P} = \frac{\sigma\varepsilon L}{V_k} Q_w , \qquad (28)$$

When metal is removed by the wheel, the metal removal rate Q_w is equal to the ratio of the metal volume to the time during which the metal volume W_{all} is removed at the table speed $V_{table.}$ The metal volume W_{all} is a metal layer with a height equal to the infeed radial feed, a length equal to the length of the machined surface, and a width equal to the grinding width (fig. 3). The grinding width can be equal or less than the height of the grinding wheel.

$$Q_w = \frac{W_{all}}{t_{table}} = \frac{S_{z,i} BL}{t_{table}} = S_{z,i} BV_{table} , \qquad (29)$$

$$\text{or at } t_{table} = \frac{L}{V_{table}} , \qquad Q_w = S_{z,i} BV_{table} , \qquad (30)$$

where t_{table} is the table pass time per workpiece length L, s.

By substituting expression (29) into equation (28), we obtain

$$A_{\Sigma P} = \frac{\sigma\varepsilon L}{V_k} S_{z,i} BV_{table} . \qquad (31)$$

Then, the grinding energy $A_{\Sigma P}$ expended by absolutely sharp grains of the wheel on plastic metal deformation in the cutting zone can be written as:

$$A_{\Sigma P} = P_{ZP} L = \frac{\sigma\varepsilon L}{V_k} S_{z,i} BV_{table} . \qquad (32)$$

Let us find the tangential component of the cutting force P_{ZP} for an absolutely sharp wheel:

$$P_{ZP} = \frac{\sigma\varepsilon S_{z,i} BV_{table}}{V_k} . \qquad (33)$$

Let us establish the relationship between the energy expended by the friction forces of blunt grains separately for a single grain and for the wheel as a whole (in which the grains have a blunting area)

$$A_{Tj} = P_{ZTj} L = \frac{\sigma b_j l_j \mu}{3} L . \qquad (34)$$

After summing the energy of all grains, we obtain the formula for the friction force energy:

$$A_{\Sigma T} = \sum_{j=1}^{J} A_{Tj} = \sum_{j=1}^{J} P_{ZTj} L = \frac{\sigma\mu}{3} L \sum_{j=1}^{J} b_j l_j = \frac{\sigma\mu}{3} L \sum_{j=1}^{J} f_j = \frac{\sigma\dot{\varepsilon} L}{V_k} F_{fr} , \qquad (35)$$

$$F_{fr} = \sum_{j=1}^{J} f_j , \qquad (36)$$

where f_j is the blunting area of a single grain, m^2; F_{fr} is the sum of the blunting areas of single grains, m^2.

$$A_{\Sigma T} = P_{ZT} L = \frac{\sigma\mu}{3}\eta B L_K L, \qquad (37)$$

$$P_{ZT} = \sum_{j=1}^{J} P_{ZTj}, \qquad (38)$$

where P_{ZT} is the tangential component of the force of cutting with the wheel arising from friction, N; η is the degree of recession of the circle [30].

$$P_{ZT} = \frac{\sigma\mu}{3}\eta B L_K, \qquad (39)$$

$$L_K = \sqrt{D S_{z,i}}, \qquad (40)$$

where L_K is the length of the contact arc for flat grinding, m; D is the diameter of the grinding wheel, m.

Then, the tangential component of the cutting force can be calculated as:

$$P_Z = \frac{\sigma\varepsilon V_{table} B S_{z,i}}{V_k} + \frac{\sigma\mu\eta B}{3}\sqrt{D S_{z,i}}. \qquad (41)$$

We similarly obtain an expression for the radial component of the cutting force:

$$P_Y = \frac{\sigma\varepsilon V_{table} B S_{z,i}}{V_k} \mathrm{tg}\beta + \frac{\sigma\eta B}{3}\sqrt{D S_{z,i}}. \qquad (42)$$

In previous studies [30], we proved that tgβ ≈ 0.68 and ε ≈ 2.8. Let us take into account the obtained values in formulas (41) and (42) to obtain:

$$P_Z = 2,8\frac{\sigma V_{table} B S_{z,i}}{V_k} + \frac{\sigma\mu\eta B}{3}\sqrt{D S_{z,i}}, \qquad (43)$$

$$P_Y = 1,9\frac{\sigma V_{table} B S_{z,i}}{V_k} + \frac{\eta B \sigma}{3}\sqrt{D S_{z,i}}. \qquad (44)$$

The adequacy of the above model of the cutting force of flat grinding was experimentally investigated in a university laboratory on a *3L722A* flat grinding machine using equipment from *KISTLER* to record and analyze cutting forces. We measured the radial component of the cutting force as the most significant indicator for processing accuracy (fig. 4). A *3L722A* flat grinding machine uses the wheel periphery for machining on a rectangular table, which corresponds to the considered flat grinding type. The technical capacity of the used *KISTLER 9257B* dynamometer meets the test requirements, while its accuracy is ± 3.5 %, which is sufficient for the measurements. Experimental verification was carried out on 500×20×20 mm (length×width×height) *steel 45* (0.45 % *C*) samples. A sensor receiving signals from two modules was used to measure the cutting force. The received digital signals are then processed on a computer. A wheel with the most common characteristic *25AF40L10V* was used. It should be noted that the developed force model makes it possible to take into account circles of various characteristics.

Fig. 4 presents the experimental results in graphs reflecting the relationship of the radial component of the cutting force with changes in various parameters of the cutting conditions. The difference between the calculated values and experimental data averaged 10 %. Thus, the adequacy of the presented model for calculating the cutting force for flat grinding is proved.

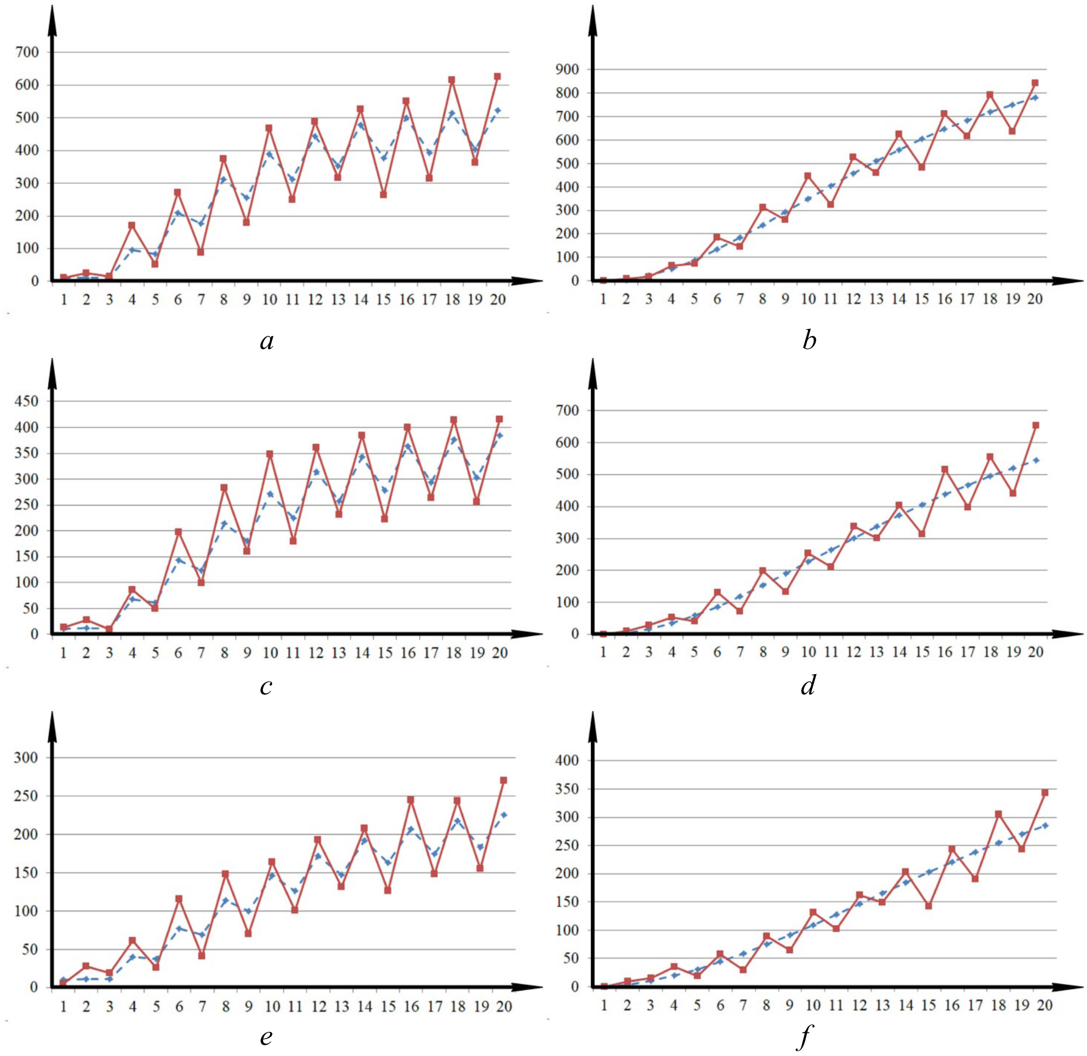


Fig. 4. The relationship between the radial component of the cutting force and the sequential number of the table pass with the workpiece at PX-XX (V_{table} = 10m/min and S_{rad} = 0.024 (*a*), 0.018 (*b*), 0.011 (*c*) m/pass) and (V_{table} = 10m/min and S_{rad} = 0.024 (d), 0.018 (e), 0.011 (e) m/pass): vertical axis on the graphs is the radial component of the cutting force, N; horizontal axis on the graphs is the sequence number of the table stroke; solid line – calculated values of cutting force; dashed line – experimental values of cutting force

Results and discussion

The resulting analytic expressions (43) and (44) are presented in engineering form, which makes it possible to practically use to calculate cutting force acting in the process of flat grinding, taking into account the cutting mode, specifications, diameter and width of the wheel, physical and mechanical properties of the workpiece, and many other parameters. The first terms of equations (43) and (44) take into account the cutting force needed for the plastic deformation of metal in the shear zone (to remove the metal). The addends take into account the cutting force needed to overcome the friction force arising from the blunting areas on the grains. The presented force model can serve as the future foundation for an analytical model for calculating cutting depth to predict changes in the technical size and its inaccuracies.

The components of the cutting force can be found in two ways. The first is based on the equality of the metal volumes removed in one pass of the workpiece by a set of single cutting grains and by the grinding wheel as a whole (as described above). The second method takes into account the equality of metal cutoff intensities from the grinding wheel and the individual cutting grains making contact with the workpiece.

Both approaches are based on the energy balance proposed by *S.N. Korchak* on the equality of the energy expended to remove metal from the workpiece surface being machined by single abrasive grains and the grinding wheel as a whole (i.e. the equality of energy in terms of the volume of metal removed and the equality in the rate of stock removal).

In a previous study [31], we considered in detail an algorithm of applying the second method to model the cutting force components based on the equal intensity of metal removal by the grinding wheel and the intensity of metal removal by cutting abrasive grains in the zone of contact with the workpiece. The equations of the cutting force model obtained by these two methods are identical. As a result, we can say that these analytical cutting force models, which are built on the assumptions and approaches from *S.N. Korchak*'s studies on the theory of cutting and plastic deformation of metal, are reliable regardless of the condition of equality of metal removal intensities or volumes.

The resulting analytical model for calculating cutting force based on the equality of the removal rate and the volumes of removed metal by the wheel as a whole and by single grains in the zone of contact between the wheel and the workpiece was experimentally confirmed, which indicates its adequacy.

Conclusions

1. The lack of adequate analytical models for calculating cutting force and depth for flat grinding in *CAM* systems from various manufacturers results in the need for manual selection of cutting conditions when an operation is designed.

2. The lack of systems for automatic calculation of cutting conditions for *CNC* flat grinding operations (digital tool for *CAM* systems) is a scientific and technical problem leading to the need to develop an analytical model of the cutting force that establishes a relationship between the force and the depth of the metal cut made by single grains with a stock removal and the cutting force that occurs during grinding with a wheel as a whole.

3. An analytical model for calculating the cutting force in flat grinding, which establishes the relationship of the cutting force with the cutting depth and the volumes of metal removed by single grains and the wheel as a whole based on the integration of microvolumes and microforces when the metal is cut by grains is proposed.

4. Mathematical modeling of the relationship between the cutting force and cutting conditions with the parameters of microcutting by a group of single grains was carried out, based on the equality of grinding energy when removing the same volume of metal.

5. The presented model of the cutting force occurring during flat grinding was confirmed experimentally and coincided with a model of cutting force which is also based on *S.N. Korchak*'s assumptions and the equal intensity of metal removal by the grinding wheel as a whole and by single grains in the contact zone of the wheel with the workpiece.

6. A practical application of the developed cutting force model will be the creation of a digital twin of flat grinding, which can predict the stability of the machined surface accuracy and quality and support optimization of designed operations, requiring the selection of the optimal cycles of cutting conditions and other input parameters (for example, the specifications of the abrasive tool), which is of great practical importance for the digitalization of engineering processes.

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Conflicts of Interest

The authors declare no conflict of interest.

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