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Selection of a finishing hole processing method for multi-product manufacturing based on solving a multi-objective optimization problem

Vadim Stelmakov [a, *], *Mikhail Gimadeev* [b]

Pacific National University, 136 Tihookeanskaya St., Khabarovsk, 680035, Russian Federation

[a] https://orcid.org/0000-0003-2763-1956, 009062@togudv.ru; [b] https://orcid.org/0000-0001-6685-519X, 009063@togudv.ru

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ABSTRACT

Introduction. In modern mechanical engineering, particularly in multi-product manufacturing, the development of hole machining processes remains a significant challenge. High requirements for form accuracy, hole axis alignment, and surface roughness necessitate the use of various finishing methods, such as boring and reaming. Furthermore, based on current research in hole machining, another promising method is emerging — milling employing different strategies. Each of these methods has unique advantages and limitations concerning achievable accuracy, productivity, and cost-effectiveness. In an environment of heightened competition and the need for rapid adaptation to market changes, manufacturers face the imperative to optimize their technological processes. This is especially critical for hole machining operations, which constitute a significant portion of the total manufacturing complexity of components. ***Subject***. This paper investigates an approach to selecting the optimal hole finishing method – boring, reaming, or milling with circular/helical interpolation – based on solving a multi-objective optimization problem. Special emphasis is placed on analyzing the influence of the number of holes being machined on the optimization results. ***The purpose of this* work** is to develop an approach for selecting a hole finishing method based on achievable form accuracy, cost, and productivity, through solving a structural optimization problem adapted to the conditions of multi-product manufacturing. **Methodology.** The approach is based on the application of mathematical statistics to construct regression models, establishing the influence of machining parameters on deviations from roundness and cylindricity for each finishing method under investigation. A genetic algorithm (*GA*) is employed to identify the *Pareto*-optimal solution set, and the “ideal point” method is used to select the final option. **Results and discussion.** Solving the optimization problem revealed that for batches containing 1 to 30 high-precision holes, milling with helical interpolation is the optimal method, whereas boring becomes optimal for batches of 40 or more holes. Recommendations regarding the selection of machining parameters for each optimal method were derived. The developed approach enables a substantiated selection of the processing method during the technological preparation stage, thereby reducing process planning complexity and enhancing economic efficiency.

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Introduction

The design and development of hole-machining processes occupy a special place among the diverse challenges of modern mechanical engineering. Increased attention to this type of process is primarily due to the functional purpose of the machined surfaces. Since these surfaces most often ensure the relative positioning of parts, assemblies, and components in machines and mechanisms, achieving precision and quality is an important task, the successful solution of which will directly impact the durability and operational reliability of the mechanism.

*** Corresponding author**
Stelmakov Vadim A., Ph.D. (Engineering), Associate Professor
Pacific National University,
136 Tihookeanskaya st.
680035, Khabarovsk, Russian Federation
Tel.: +7 962 221-74-60, **e-mail:** 009062@togudv.ru

Traditionally, high-precision hole machining is carried out in three main stages – roughing, semi-finishing, and finishing – using different equipment configurations, specific types of cutting tools, and auxiliary equipment. Various studies are devoted to the issues of designing technological machining processes, including those for high-precision holes [1–6]. An analysis of the aforementioned works shows that achieving the required parameters of hole accuracy is a labor-intensive operation requiring the use of a variety of cutting tools (drills, reamers, boring cutters, countersinks), which, in turn, increases the labor intensity of process design.

However, the conditions of market relations, coupled with the intensification of competition between enterprises in the mechanical processing industry market, require the latter to quickly and adequately respond to changes in market conditions, to reduce the time for designing technological projects, as well as for manufacturing products. This is directly related to the desire to create competitive and customized products with increased demands on precision and quality of manufacture. In this regard, enterprises with a multi-product production process find themselves in a situation where they need to maintain the ability to produce a wide range of products while remaining competitive by reducing the cost and labor intensity of design operations for creating processing routes. Under these conditions, the use of traditional hole-processing routes does not always satisfy the requirements for speed, cost, and adaptability of the production process.

From this point of view, it is necessary to consider the current level of technological development from several positions. Firstly, there is the multifunctionality of metal-cutting equipment and the capabilities of modern *CNC* systems. Secondly, the use of multifunctional cutting tools. Thirdly, the capabilities of modern *CAD*/*CAM* systems. These three aspects are keys to a multi-product manufacturing process.

Multifunctional *CNC* machining centers allow for the machining of complex, box-shaped parts with a reduced number of setups and a wider range of machined surfaces. However, due to economic factors, using specialized tooling for holemaking is not always practical, as it increases the tool inventory, overloads the machine's tool magazine, and increases setup time. Modern trends necessitate the use of multifunctional cutting tools capable of machining a wide range of surfaces with varying dimensions. This necessitates the proper selection of machining methods and, crucially, machining strategies. For example, drilling can be replaced with helical interpolation milling, thereby preparing the holes for finishing operations. This aspect is also important for achieving the specified precision and quality parameters of the machined surfaces.

Hole accuracy parameters include diameter, shape, axis position, and surface roughness. Geometrical accuracy refers to macroscopic deviations, which for holes are typically standardized by roundness and cylindricity tolerances. This study examines high-precision holes with a diameter accuracy that meets quality standards *IT6–IT7*. In this case, the error of the geometric shape, as a rule, does not exceed 0.3–0.5 of the tolerance for the diametrical size, the tolerance for the axis location is within 0.008–0.025 mm, and the surface roughness is $\boldsymbol{Ra}$ = 1.1–1.6 μm. Ensuring the specified parameters of shape accuracy is the task of the finishing stage, for which methods such as reaming and boring are traditionally used. However, studies [7–15] describe approaches to the application of various milling strategies with the aim of determining their place in machining routes. The kinematics and dynamics of the milling process are analyzed, and an assessment is provided of the achievement of diametrical dimensions and the location of the hole axis.

In the work of the authors [7], it has been shown that milling with helical interpolation is a highly effective alternative to traditional drilling, providing a significant improvement in the quality of holes due to the unique kinematics of the process, which combines axial feed and radial movement of the tool. In the work of the authors [8], it has been shown that the deviation of the diametrical size during milling with helical interpolation is a consequence of the deformation of the tool under the action of the radial cutting force. The authors found that the choice of milling direction also affects the final diameter size (with up-milling, the hole diameter exceeds the nominal one, while with down-milling, it is smaller). Based on the analysis of kinematics and force impact, the authors developed a predictive model that allows for the highly accurate determination of the hole diameter at any depth, depending on the cutting modes and milling direction.

Based on geometric modeling, it was established in [9] that the ratio of the volumes of material removed by the end and peripheral parts of the cutting edges of the cutter is a constant value, depending solely on the ratio of the diameters of the hole being machined and the tool, and is independent of the cutting modes. This relationship provides the basis for developing force-action models to compensate for tool path deviations and optimize the process. The authors [10] explore the use of helical interpolation milling for machining carbon fiber-reinforced plastics and titanium alloys.

The authors' work [11] presents a numerical simulation and experimental study of cutting forces during milling with helical interpolation. The primary focus is on developing a mathematical model of cutting forces that takes into account the simultaneous action of both the peripheral and end cutting edges of the tool. The model also takes into account the kinematics of the process associated with spindle rotation and the helical trajectory of the tool. Experiments confirmed the adequacy of the model, demonstrating good agreement with the results of numerical simulation (with an error of approximately 10%).

In the work of the authors [12], the milling of holes with helical interpolation in composite materials *SiCp*/*Al* is investigated. The work found that the quality of the hole surface improves with an increase in rotation frequency (roughness decreases by 17.13% with an increase from 500 to 2,000 rpm) and worsens with an increase in the pitch of helix (roughness increases by 68.86% with an increase in pitch from 0.1 to 0.4 mm). In the work of the authors [13], a mathematical model was developed that takes into account the relationship between cutting force and vibrations during milling with helical interpolation. The authors [14] developed a mathematical model for calculating cutting depths and volumes of material removed. The model takes into account the division of the cutting zone into several regions depending on the variation of such parameters as the helical pitch (a_p) and eccentricity (e).

The authors [15] presented a study devoted to modeling the surface topography obtained during hole machining using helical milling. The paper developed a methodology for constructing a three-dimensional simulation of the surface formed after machining with a cylindrical end mill, taking into account the tool kinematics and its runout error.

The authors [16] investigated the process of forming cylindrical holes during finish milling with circular interpolation. It was found that the shape accuracy significantly depends on the tool penetration parameters (radius, angle, and feed rate) and the elastic deformations of the cutting tool. Based on experimental studies and mathematical modeling, recommendations are given for selecting each of the parameters. The developed regression model and the elastic deformation model of the tool make it possible to predict deviations from roundness and cylindricity with an accuracy of up to 15%.

Thus, the milling method with different strategies has the following advantages: versatility (an end mill of one nominal diameter can produce holes of different diameters), and, taking into account the selection of machining modes, the ability to ensure high productivity. The main drawback of this method is the limitation on machining depth. Given the diversity of studies devoted to hole milling, the applicability of these methods (strategies) for finishing in terms of achievable accuracy and productivity remains insufficiently studied.

The boring method is characterized by a wide variety of tool designs, the choice of which depends on the diameter and depth of the hole being machined. A distinctive feature of boring tools is the ability to precisely adjust the position of the cutting edge, which enables high machining accuracy. As noted in [17, 18], chip control is a critical factor in boring. In [17], an analytical model for calculating cutting forces was developed that takes into account the tool trajectory and chip destruction process. The formation of short and thick chips leads to increased cutting force and vibration, while long chips, when accumulated, degrade the surface quality of the hole.

In finishing operations, the risk of vibration caused by incorrectly selected cutting insert geometry, particularly the value of the side cutting edge angle and the nose radius, becomes a significant factor. Increasing the nose radius, as shown in studies [19–21], leads to an increase in the radial component of the cutting force and the magnitude of the elastic deflection of the system. This, in turn, causes vibrations, reducing accuracy and degrading surface quality.

Thus, the main advantages of boring include the ability to precisely adjust to the final size and ensure high surface quality. The main disadvantage is the high cost of the tool, which becomes a key economic constraint when it is used in multi-product manufacturing.

The reaming method is highly productive. The quality of the reaming operation is largely determined by the condition of the pre-machined hole [4–6]. This method improves the surface quality of the machined hole.

Unlike boring tools, reamers allow for higher feed rates and do not require time-consuming adjustment to the final diameter on the machining center. However, while ensuring high diameter accuracy, reaming does not allow for correction of hole axis deviations. As established in works [22–24], the misalignment of the machine spindle and the hole being machined significantly affects the radial component of the cutting force, which, in turn, leads to significant deviations from roundness. Therefore, during the reaming process, the axis positioning accuracy may not only fail to improve but may even deteriorate. Another important factor, especially for multi-product manufacturing, is the highly specialized nature of reamers: each diameter requires a separate tool, which increases the number of reamers and tooling costs.

Based on the above, it can be concluded that each finishing method has its advantages and disadvantages. For example, boring allows for high accuracy of diameter size, shape, and axis position, but tool cost and setup complexity will play a key role in its selection. Milling with circular and helical interpolation strategies can provide both high productivity and high accuracy. Reaming in finishing operations allows for high quality and productivity but has several disadvantages related to the lack of versatility for various hole diameters.

In summary, it can be said that the choice of hole machining method on *CNC* machining centers depends on many factors, and one of the most important of these, in addition to the achieved accuracy and quality indicators, is the economic component of the cost of machining (including the depreciation of tooling, the cost per hour of equipment operation, the cost of cutting tools and components) and productivity. For multi-product manufacturing, the use of high-performance, expensive tooling is not always practicable, so milling is a viable alternative. However, given the diversity of solutions developed to date, the overall approach to developing a machining route, selecting machining methods and strategies to achieve specified precision parameters in a multi-product manufacturing environment requires further development.

Thus, ***the purpose of this work*** is to develop an approach to selecting a method for finishing holes based on the achieved shape accuracy, cost, and productivity, by solving the problem of structural optimization as applied to the conditions of multi-product production.

Based on the stated objective of the study, it is clear that we will solve the optimization problem based on a set of optimality criteria (more than two); therefore, the problem being solved relates to multi-criteria optimization.

From a decision-making strategy perspective, there are three main types of multi-criteria optimization problems. The first type considers the prioritization of some optimality criteria over others; these types of problems are solved using the lexicographic method. In the second type, the optimality criteria are combined into a single function of the form:

$$F = a_1 \cdot f_1(x) + a_2 \cdot f_2(x) + ... + a_n \cdot f_n(x) \rightarrow \max/\min,$$

where a_1, a_2 … a_n are weighting factors; $f_1(x)$, $f_2(x)$ … $f_n(x)$ are optimality criteria. Moreover, $a_1 + a_2 + \ldots + a_n = 1$.

Problems of this type are solved using the weighted sum (linear convolution) method. The third type of problem is characterized by the equivalence of optimality criteria; such problems are solved by finding the *Pareto* frontier.

It is impossible to classify the multi-criteria optimization problem being solved into the first two types, since certain priorities regarding optimality criteria must be established before the optimization process begins. The third type of problem is used when it is impossible to determine priorities and the optimality criteria are equivalent, a condition that corresponds to the problem being solved.

In the works of the authors [25–26], a review of population methods of *Pareto* approximation is made, which use evolutionary algorithms (for example, genetic algorithms (*GA*), particle swarm method) to find a set of optimal values according to *Pareto*. These multi-criteria optimization methods, based on constructing a *Pareto* set approximation, constitute a promising and actively developing class of approaches to solving such problems, particularly with the use of a hybrid approach. In [26], the primary focus is on software products and algorithms for assessing the quality of *Pareto* approximations.

In the work of the authors [27], a solution to the problem of multi-criteria parametric optimization of the process of milling pockets from an aluminum alloy *Al 6061-T6* is shown. This paper demonstrates the effectiveness of a hybrid approach employing regression analysis and a genetic algorithm to determine optimal machining conditions. The authors of [28] address a multi-criteria optimization problem aimed at reducing energy consumption and specific cutting energy during milling. The approach is based on a hybrid model that combines an artificial neural network trained using backpropagation and the NSGA-II genetic algorithm as a predictive model. The best solution from the Pareto frontier was selected using the "ideal point" method. The authors demonstrated that the optimized parameters enable a significant reduction in energy consumption and specific energy input for cutting compared to the design conditions.

The authors [29] examine multi-criteria optimization from the perspective of determining cutting conditions for the roughing and finishing stages of machining as a single, interconnected system. The optimality criteria chosen in the study were minimization of main time, energy consumption, and surface roughness. The study demonstrates that joint optimization of conditions at all stages of machining is more effective than optimizing each stage separately.

An analysis of the results of modern research allows us to conclude that the most promising are hybrid methods of multi-criteria optimization, which combine predictive models used for the mathematical description of objective functions, genetic algorithms for searching for the Pareto front, and methods for determining the optimal point of the criterion space (the "ideal point" method and the control indicator method). Therefore, to achieve our goal, we will use a hybrid multi-criteria optimization model, employing a genetic algorithm as the most universal method suitable for solving problems with equivalent optimality criteria. To reduce the number of optimal variants obtained, we will use the "ideal point" method.

Thus, the following ***tasks*** will be solved during the study:

1. To develop mathematical models describing the influence of technological processing parameters on deviations from roundness and cylindricity during finishing of holes using various methods (boring, reaming, milling with circular and helical interpolation).

2. To develop a multi-criteria optimization model and determine the parameters of a genetic algorithm for finding a set of Pareto optimal values.

3. To analyze the influence of the batch size of machined holes on the optimization results (*Pareto* frontier positions), and to determine practical recommendations for choosing a method for finishing holes in conditions of multi-product production.

Methods

Experimental studies were conducted on *DMG MORI* (Germany) milling machining centers with a *Heidenhain TNC 620 CNC* system: a three-axis *DMC 635V ecoline* and a five-axis *DMU 50 ecoline*. The maximum spindle speed was 8,000 rpm, and the maximum driver speed was 24 m/min.

The cutting tool was inspected and measured using a *Heidenhain TT140* optical touch probe. A *Heidenhain TS 640* measuring probe was used to measure the diameters and center coordinates of machined holes in three sections.

Deviations from roundness and cylindricity were measured on a *Roundcom-41C* measuring device. The material chosen for machining was aluminum alloy *D16T*, which is widely used in the aircraft and automotive industries due to its physical and mechanical properties. The machining depth for each of the methods was ***b*** = 17 mm. The diameter of the machined holes was chosen to be 13.5 mm. This decision was aimed at comparing the efficiency of the machining methods themselves and ensuring the accuracy of a detailed economic cost model, the parameters of which depend significantly on the specific size. The main process parameters are presented in Table 1.

For the milling method (with circular and helical interpolation), the choice of a 10 mm diameter, two-flute, solid-carbide end mill was driven by the desire for tool standardization in a multi-product manufacturing environment, as well as the need to minimize variable factors during a comparative analysis of strategies. Furthermore, preliminary calculations showed that, taking into account the acceleration dynamics of the drivers, the use of this cutter allows investigation of the full range of feed rates for various strategies, up to the maximum values.

Table 1

Planning of experimental studies

Processing method	Feed rate, (mm/tooth); (rpm)	Pitch of the helical trajectory a_p (mm)	Cutting speed V (m/min)	Tool support
Circular interpolation milling	0.03	–	220	Solid carbide two-tooth end mill with a diameter of 10 mm (*DIN 6528*) from *Sandik Corommand*
	0.05			
	0.07			
Helical interpolation milling	0.03	0.3		
		0.7		
		1.1		
	0.05	0.3		
		0.7		
		1.1		
	0.07	0.3		
		0.7		
		1.1		
Boring	0.05	–	40	Boring bar *C5-391.37A-16 070 A*, and carbide boring cutter *R429U-E16-11066TC06* from *Sandvik Corommand*
	0.075			
	0.1			
Deployment	0.15	–	8	*Walter* 13.5mm diameter straight flute reamer (*DIN 212*)
	0.3			
	0.45			

Results and Discussion

The vector describing the solution space will be the processing methods: milling with circular interpolation, milling with helical interpolation, boring, and reaming:

$$\vec{x}_k = \{x_1, x_2, x_3, x_4\}, \tag{1}$$

where x_1 is circular interpolation milling; x_2 is helical interpolation milling; x_3 is boring method; x_4 is deployment method.

At the first stage, it is necessary to formalize the optimization problem, which consists of choosing the optimal processing method based on minimizing the indicators of shape deviation (roundness and cylindricality), the cost of processing while achieving maximum productivity. Thus, taking into account (1), the optimization problem in general looks like this:

$$\min_{x\in\theta}\left\{f_r(x_k), f_{cyl}(x_k), f_c(x_k)\right\};\ \max_{x\in\theta}\left\{f_{pr}(x_k)\right\},$$

where $f_r(x_k)$ and $f_{cyl}(x_k)$ are the objective functions for roundness and cylindricity deviation (μm); $f_{pr}(x_k)$ is the objective function of machining productivity (holes/min); $f_c(x_k)$ is the objective function of processing cost (rub); θ is the experimental space.

In the second stage, it is necessary to determine the objective functions for each optimality criterion. The objective functions for the roundness and cylindricity parameters are determined using mathematical statistics methods. To this end, experiments were conducted, the results of which are presented in Table 2.

The methodology for statistical processing of experimental data included the following stages: data preparation, testing for homogeneity of variances, and regression analysis. At the data preparation stage, for each of the processing methods (milling with circular interpolation, boring, reaming, and milling with helical interpolation), the arithmetic mean values and variances of the parameters (deviations from roundness and cylindricity) were calculated for five repeated experiments at each point of the experimental plan.

Table 2

Results of experimental studies

Processing method	Feed rate, (mm/tooth); (rpm)	Pitch of the helical trajectory a_p (mm)	Average value of deviation from roundness (µm)	Average deviation from cylindricity (µm)	Variance of roundness deviations (µm^2)	Variance of cylindricity deviations (µm^2)
Circular interpolation milling	0.03	–	22.2	34.1	4.270	4.170
	0.05		25.5	38.4	1.323	4.123
	0.07		30.5	44.9	0.503	4.163
Helical interpolation milling	0.03	0.3	6.96	15.1	0.083	0.177
		0.7	7.47	16.3	0.383	0.548
		1.1	7.98	17.5	0.373	0.423
	0.05	0.3	7.66	13.3	0.277	0.163
		0.7	7.90	15.6	0.420	0.658
		1.1	8.14	17.9	0.334	0.247
	0.07	0.3	8.53	11.4	0.115	0.277
		0.7	8.44	14.7	0.378	0.465
		1.1	8.34	18.3	0.145	0.307
Boring	0.05	–	5.8	9.4	0.655	1.563
	0.075		5.7	11.1	0.002	1.363
	0.1		6.3	11.5	0.861	5.743
Deployment	0.15	–	8.87	14.2	0.49	1.083
	0.3		10.5	15.2	1.86	1.123
	0.45		9.0	17.3	0.39	1.243

In total, 90 experiments were carried out in the study according to the plan presented in Table 2. When testing for homogeneity of variances, the confidence interval for estimating parameters was taken for a significance level of $\boldsymbol{p} = 0.05$ (confidence probability 95%). The test was performed using *Cochran's G*-test. Experimental data were verified within each processing method and for each controlled accuracy parameter (roundness and cylindricity). The results of the homogeneity of variance test are presented in Table 3.

Thus, for all tested groups, the condition $\mathbf{G_0} < \boldsymbol{G_{cr}}$ is satisfied; the null hypotheses about the homogeneity of variances are not rejected. Therefore, the variances of measurements within each method and for each parameter are statistically homogeneous, which makes it possible to apply the regression analysis method. For methods where a single factor was varied, a univariate regression analysis was used. For the helical

Table 3

Results of homogeneity of variance tests

Processing method	Observed value *Go:* deviation from roundness	Observed value *Go:* deviation from cylindricity	Critical value G_{cr}
Circular interpolation milling	0.7007	0.3348	0.707
Helical interpolation milling	0.1675	0.2015	0.352
Boring	0.5669	0.662	0.707
Deployment	0.6788	0.360	0.707

interpolation milling process, characterized by the influence of two factors, a multivariate regression analysis was performed. The general form of the regression model is presented as follows:

$$f(x) = b_0 + \sum_{i=1}^{k} b_i \cdot x_i + \sum_{\substack{i=1 \\ j\neq 1}}^{k} b_{ij} \cdot x_i \cdot x_j + \ldots + \sum_{i=1}^{k} b_{ij} \cdot x_i^n + \varepsilon, \quad (2)$$

where $f(x)$ is the response parameter value; $\boldsymbol{b_0 \ldots b_{ij}}$ are the regression coefficients; $\boldsymbol{x_i}$ and $\boldsymbol{x_j}$ are the independent factors; $\boldsymbol{k}$ is the number of factors; $\boldsymbol{\varepsilon}$ is the regression error.

In the next step, the regression coefficients and their confidence intervals were determined, and their statistical significance was verified. The residual variance was calculated, and the model adequacy was tested using *Fisher's* exact test (F-test). Thus, in accordance with equation (2), the following objective functions were obtained for each method:

$$f_r(x_k) = \begin{cases} 15.65 + 209 \cdot F_z, & \text{for } x_1 \quad (F_o = 0.47 < F_{cr} = 4,75) \\ 5.22 + 45.125 \cdot F_z + 2.29 \cdot a_p - 33.75 \cdot F_z \cdot a_p, & \text{for } x_2 \quad (F_o = 0.1 < F_{cr} = 7.16) \\ 4.629 + 15.6 \cdot F_u, & \text{for } x_3 \quad (F_o = 0.1 < F_{cr} = 4.75) \\ 9.48 + 0.46 \cdot F_u, & \text{for } x_4 \quad (F_o = 2.5 < F_{cr} = 4.75) \end{cases}$$

$$f_{cyl}(x_k) = \begin{cases} 25.59 + 271.5 \cdot F_z, & \text{for } x_1 \quad (F_o = 0,39 < F_{cr} = 4,75) \\ 18.1 - 131.4 \cdot F_z + 1.06 \cdot a_p - 136.25 \cdot F_z \cdot a_p, & \text{for } x_2 \quad (F_o = 0.1 < F_{cr} = 7.16) \\ 7.59 + 41.2 \cdot F_u, & \text{for } x_3 \quad (F_o = 0.13 < F_{cr} = 4.75) \\ 12.49 + 10.33 \cdot F_u, & \text{for } x_4 \quad (F_o = 0.39 < F_{cr} = 4.75) \end{cases}$$

The productivity objectives for each method were determined based on the kinematics of the machining process. In circular interpolation milling, the process involves an auxiliary positioning motion to bring the tool to a specified depth, followed by a shaping motion and stock removal along a circular path. When milling with helical interpolation, the tool plunging and stock removal occur along a helical trajectory; the shaping motion is formed by moving the axes of the machining center along three coordinate axes ***X*, *Y*, *Z*** simultaneously. The kinematics of boring and reaming involve sequential tool feed along a single ***Z***-axis. Therefore, the productivity objective functions for calculating the number of holes produced per minute are described by the following equations:

$$f_{pr}(x_k) = \begin{cases} \left(\dfrac{\pi \cdot (D_h - D_t)}{z \cdot F_z \cdot n} \right)^{-1}, & \text{for } x_1 \\ \left(\dfrac{\pi \cdot (D_h - D_t) \cdot \dfrac{l_h}{a_p}}{z \cdot F_z \cdot n} \right)^{-1}, & \text{for } x_2 \\ \left(\dfrac{l_h}{F_u \cdot n} \right), & \text{for } x_3 \text{ and } x_4 \end{cases}$$

where $\boldsymbol{D_h}$ is the diameter of the hole being machined (mm); $\boldsymbol{D_t}$ is the tool diameter (mm); $\boldsymbol{l_h}$ is the length of the hole being machined (mm); $\boldsymbol{z}$ is the number of teeth.

The objective cost function, taking into account the cost of equipment operation, energy, and tool costs, and considering the number of holes machined [31], is:

$$f_c(x_k) = t_0 \cdot E + t_0 \cdot C_{es} + \frac{t_0 \cdot E_u(x_k)}{T} + \frac{t_{pf} \cdot E}{Q},$$

where t_0 is the regular time (min); E is the cost per minute of machining center operation (rub/min); C_{es} are energy costs per minute of cutting (rub/min); T is tool life, min; E_u are tool costs for the service life (rub); t_{pf} is preparatory and final time (min); Q is the number of holes to be processed (pcs).

We will find the costs of electricity (C_{es}) and the costs of the tool (E_u):

$$E_u\left(x_k\right)=\begin{cases}C_{dn}+T_{cm}\cdot E_c, & \text{for } x_1,x_2,x_4\\ \dfrac{C_{dn}\cdot f_d}{m_p\cdot q_p}+\dfrac{C_{pn}\cdot K_{ub}}{m_p}+T_{cm}\cdot E_c, & \text{for } x_3\end{cases},$$

$$C_{es}=\frac{C_e\cdot N}{60},$$

where C_{dn} is the cost of the tool (rub); f_d is a coefficient taking into account additional costs for repairing the holder; m_p is the number of cutting edges on the plate; C_{pn} is the cost of cutting insert (rub); K_{ub} is the random attrition rate; T_{cm} is the time spent on changing tools; q_p is the number of plates for which the toolholder life is calculated; E_c is the machine operator's per-minute wage rate with overhead costs; C_e are energy costs (rub/kWh); N is cutting power (kWh) (determined for each processing method).

Taking into account the provisions presented in Reference [31], and considering regional specifics, the following values are adopted: E = 2,500 rub/hour, E_c = 11.6 rub/min, C_e = 7.35 rub/kWh, q_p = 130, K_{ub} = 1.2, f_d = 1.3, and m_p = 3.

At the third stage, it is necessary to define the main constraints within which the solution to this optimization problem will lie:

$$\begin{cases}f_{\min}\le f\le f_{\max}\\ a_{p\ \min}\le a_p\le a_{p\ \max}\end{cases}=\begin{cases}30\,\dfrac{\text{mm}}{\text{min}}\le f\le 1{,}200\,\dfrac{\text{mm}}{\text{min}}\\ 0.3\ \text{mm}\le a_p\le 1.1\ mm,\end{cases}$$

where f is the feed rate (mm/min).

The first constraint that is important is the acceleration limits of the servo drives of machining centers (f_{max} = 1,200 mm/min) for such hole diameters. The helix pitch is limited to 1.1 mm due to the limited increase in productivity as productivity increases. The diameter range was kept small, as a wide range would complicate cost calculations.

After formulating the optimization problem and defining all the objective functions with constraints, we move on to the fourth stage: setting up a genetic algorithm to search for the *Pareto* set. Let's represent processing methods as individuals. Each individual is encoded by a single chromosome. The genes in this case will be represented by the main technological parameters (F_z, F_u, a_p) used in regression models, and one gene will be responsible for a key feature of the method, such as the processing strategy (circular and helical interpolation). We set the population size to 100 individuals to ensure good diversity. The fitness of each individual is then assessed based on the objective functions and constraints, after which the *GA* proceeds to the reproduction stage.

Reproduction consists of three main steps: selection, crossover, and mutation. Selection is responsible for selecting the parental individuals. Individuals with better performance have a higher probability of being selected for "reproduction"**,** which directs evolution towards improving solutions. Crossover is an important genetic operator because it generates a new chromosome by combining the genetic material of two parents. For our research, we selected a simulated binary crossover (*SBX*) with a crossover probability of 0.9. In *SBX*, the parent chromosomes are cut at a randomly chosen point, and the offspring chromosome is formed from the beginning of the first parent chromosome and the end of the second. The final step is mutation – an operator that randomly changes the genes in the offspring chromosome. The polynomial mutation (*PM*) operator is traditionally used in conjunction with *SBX*. The number of calculation iterations (generations) is set to 80. The *GA* is implemented in *Python*.

The fifth stage involves generating *Pareto*-optimal solutions using the *GA* for different numbers of machined holes (1, 10, 20, 30, 40, and 50). The convergence graphs of the *GA* for each optimality criterion are shown in Fig.1.

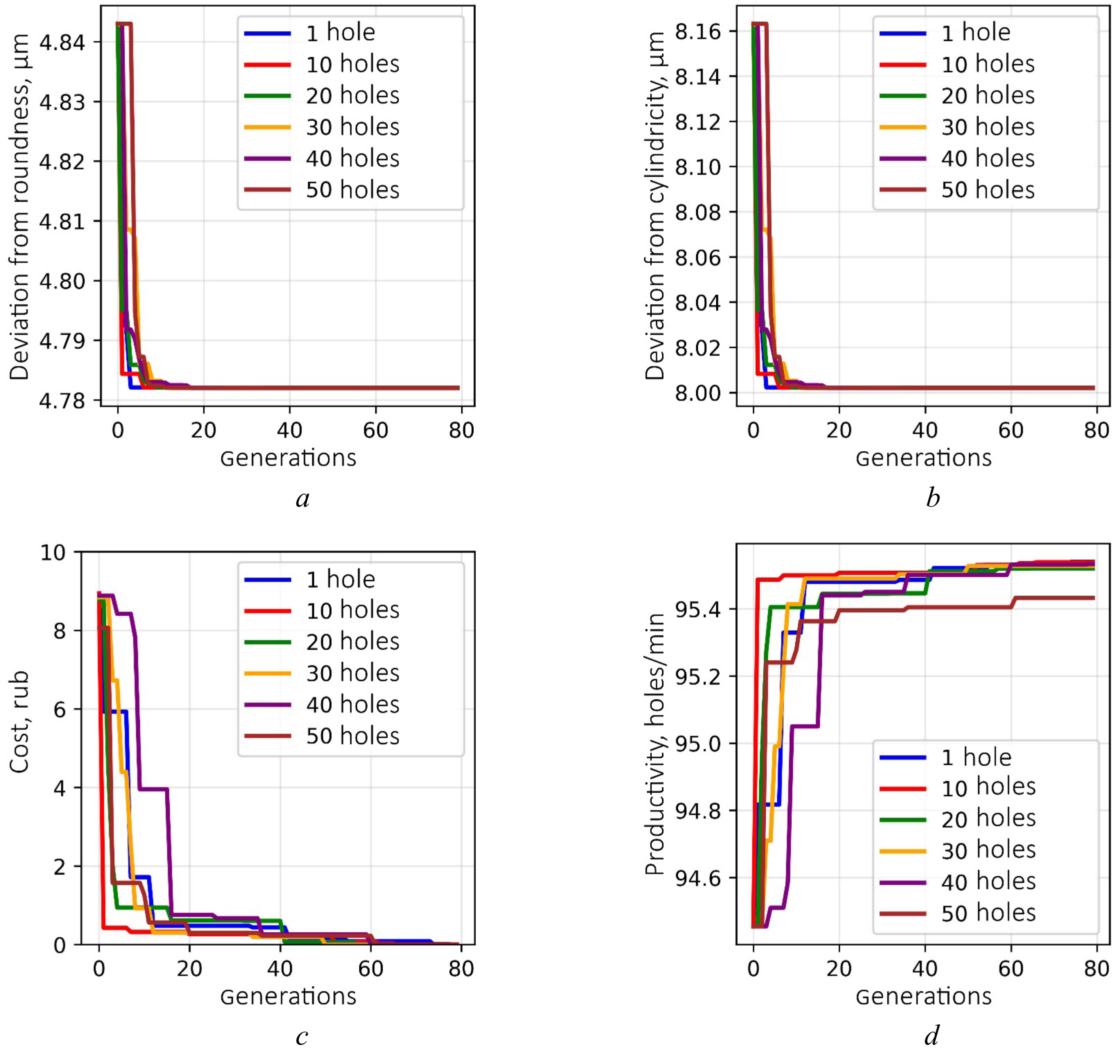

Fig. 1. Convergence plots for the optimization criteria:
a – roundness deviation; *b* – cylindricity deviation; *c* – cost; *d* – productivity

Due to the large absolute spread of processing cost values (from hundreds down to tens of rubles depending on the batch size), normalized values are presented for visualizing the convergence of the genetic algorithm for this criterion (Fig. 1, *c*): $(\boldsymbol{C}/\boldsymbol{C}_{\boldsymbol{0}_{\mathbf{min}}})\cdot 10^3$, where $\boldsymbol{C}_{\boldsymbol{0}_{\mathbf{min}}}$ is the minimum cost in the initial population for the corresponding batch size. Thus, from the analysis of the presented graphs, it can be concluded that the *GA* achieves the best values for all optimality criteria: for the criterion of deviation from roundness and cylindricity by the 20th generation; for cost by the 70th generation; and for productivity by the 60th. It can also be concluded that the selected number of generations is sufficient to solve the optimization problem.

In the sixth step, we use the "ideal point" method to find the "best" solution in the criteria space. We determine the distance for each *Pareto* point and analyze its proximity to the "ideal point" using the formula [28]:

$$S_i = \sqrt{\sum_{j=0}^{k}\left(\frac{f_i(x_k) - f_j^{\max}}{f_j^{\max}}\right)^2} \rightarrow \min, \quad (3)$$

where $\boldsymbol{S_i}$ is a global criterion that minimizes the distance to the "ideal point"; $\boldsymbol{f_j(x_i)}$ is the value of the $\boldsymbol{j}$ criterion; $\boldsymbol{f_j^{max}}$ is the maximum value of the j criterion.

Thus, using (3) on the *Pareto* set, it is possible to determine the optimal processing methods for different numbers of holes being machined (Fig. 2). The simulation results are also presented in Table 4.

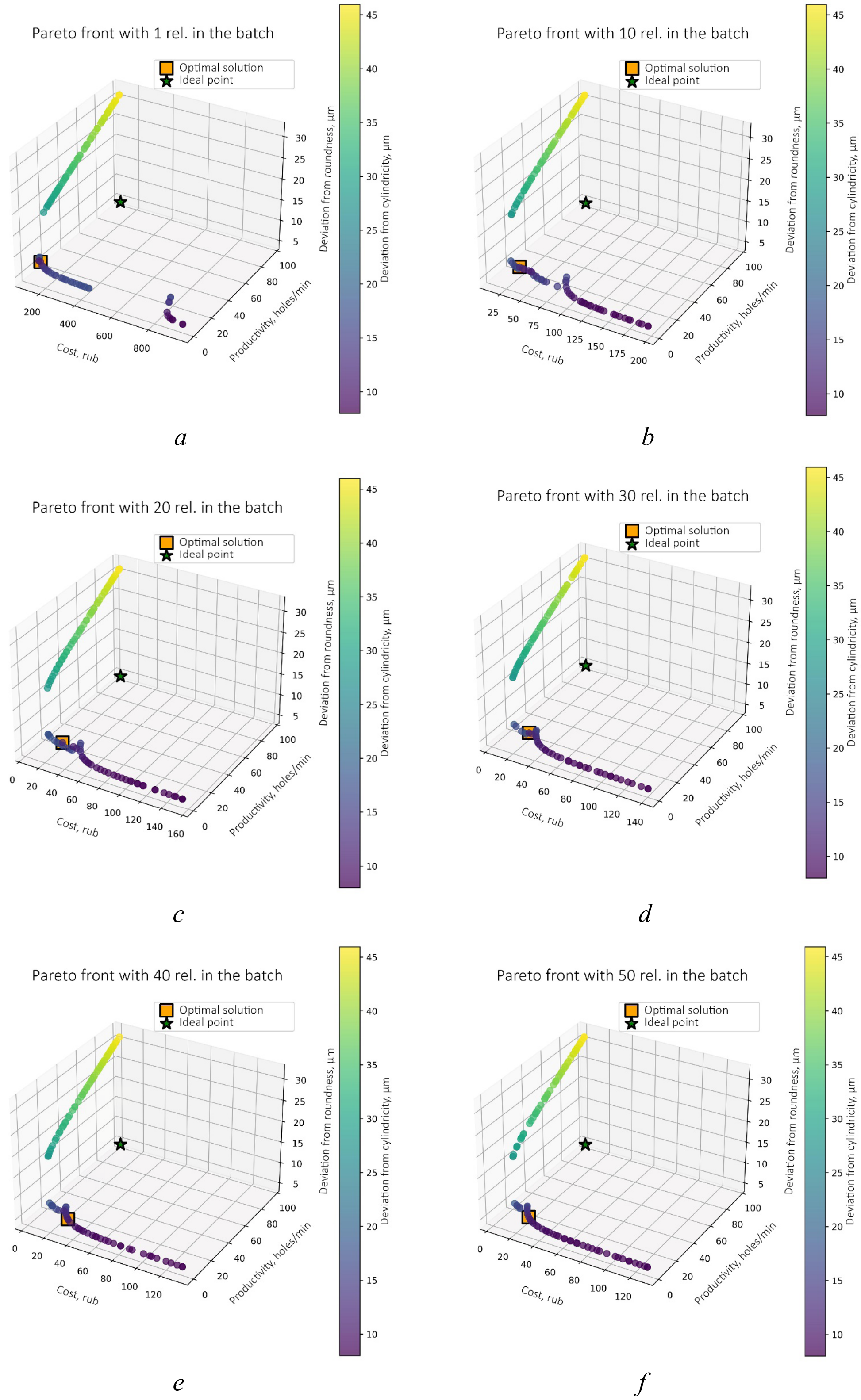

Fig. 2. Pareto fronts with optimal and ideal points indicated in the criterion space for a batch containing:
a – 1 hole; *b* – 10 holes; *c* – 20 holes; *d* – 30 holes; *e* – 40 holes; *f* – 50 holes.

An analysis of the Pareto front graphs and the results presented in Table 4 shows that helical interpolation milling is the optimal method for machining high-precision holes when the number of holes in a batch ranges from 1 to 30. As the number of holes increases, boring becomes the optimal method in terms of achievable accuracy and a slight increase in cost.

Table 4

Simulation results

Number of holes, pcs	Processing method	Feed rate (mm/teeth)	Pitch of helix (mm)	Value $f_r(x_k)$ (µm)	Value $f_{cyl}(x_k)$ (µm)	Value $f_c(x_k)$ (rub)	Value $f_{pr}(x_k)$ (holes/min)
1	Helical interpolation milling	0.075	0.3	8.53	11.01	167.64	1.69
10		0.075	0.48	8.49	12.66	39.14	2.70
20		0.075	0.42	8.50	12.18	36.17	2.40
30		0.075	0.44	8.48	12.39	33.06	2.49
40	Boring	0.10	-	6.19	11.72	32.04	4.72
50		0.11	-	6.35	12.14	26.85	5.19

Thus, the solution of a multi-criteria optimization problem allows us to determine the optimal hole machining method that ensures a balance between high precision, productivity, and low cost.

Conclusion

This paper proposes an approach to selecting a finishing method for high-precision holes in a high-variety production environment. The approach is based on solving a multi-criteria optimization problem using mathematical statistics, a genetic algorithm, and the **"**ideal point**"** method. The following results were obtained by solving the multi-criteria optimization problem:

1. Relationships were established in the form of regression models between process parameters and roundness and cylindricity deviation values for four hole finishing methods: boring, reaming, and milling with circular and helical interpolation. The developed models were validated using *Fisher's* exact test **(*F*-**test**)**.

2. Based on *Pareto* front analysis, optimal machining methods were determined depending on the batch size. It was found that for batches containing 1–30 high-precision holes, the most optimal method is helical interpolation milling with machining modes (F_z = 0.075 mm/tooth, a_p = 0.3–0.5 mm). When the number of holes increases to 50 and above, the advantage shifts to boring with modes (F_u = 0.1 mm/rev, n = 800 rpm), since a slight increase in cost is compensated by achievement of higher accuracy indices. The developed approach has practical potential for implementation in the production technological preparation system when designing machining routes.

3. Despite the promising results, this study is currently limited by the use of two machining centers and fixed cutting process conditions, including a small range of hole diameters, hole depths, etc., which undoubtedly impacts the coefficients of the regression models. Future research will expand the scope to cover a wider range of machining conditions and include additional parameters in the optimization problem, such as surface roughness and tool wear.

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Conflicts of Interest

The authors declare no conflict of interest.

Author Contributions Statement:

Stelmakov V.A. – conceptualization, experimental design, conducting experiments; writing – original draft and conclusions, data visualization (graphical representation of results), literature search and review of domestic and international sources;

Gimadeev M.R. – critical analysis and interpretation of data (contributing to conclusions), participation in manuscript discussion, reviewing and editing the manuscript text.