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Theoretical justification for calculating the absolute coordinates of a tool for dimensional setting of CNC machines to enable untended technologies

Alexandra Akintseva [a, *], *Pavel Pereverzev* [b]

South Ural State University (National Research University), 76 Lenin Ave., Chelyabinsk, 454080, Russian Federation

[a] https://orcid.org/0000-0001-8881-8461, akintsevaav@susu.ru; [b] https://orcid.org/0000-0002-1416-8918, pereverzevpp@susu.ru

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ABSTRACT

Introduction. The key obstacle to the development of “smart” mechanical engineering enterprises and the implementation of untended technologies lies in the “digital divide” (i.e., the gap between fully digitized process planning and the empirical setting of tool coordinates) during the dimensional setting of *CNC* machine tools. The design contour of pre-production engineering (*PPE*) has been completely digitized, but tool setup coordinates are still calculated empirically using trial runs because there is no theory describing the behavior of the non-stationary tolerance zone for a batch of workpieces in the absolute coordinate system of the machine. ***Objective.*** To theoretically substantiate the existence and uniqueness of the absolute coordinate of a cutting tool, guaranteeing that every workpiece in the batch, starting from the very first one, is machined within tolerance under the influence of multicomponent errors. **Methods.** The study is based on the systems analysis of technological dimensional chains and elasticity theory. The key methodological approach is axiomatic; four basic axioms are formulated (on the batch of parts, stationarity of the machine coordinate system, non-stationarity of the tolerance zone, and the generation of geometry). These axioms serve as the foundation for rigorous proofs of subsequent theorems. **Results.** Based on the adopted axioms, a theorem on the existence of a stationary “0–0 boundary” in the *CNC* coordinate system was formulated and proven for the first time. This boundary serves as a stationary reference datum linking the position of the non-stationary tolerance zone with the rigid machine coordinate system. Its uniqueness as a condition for minimum guaranteed stock removal is proven. An equation for the balance of tolerance and multicomponent errors is derived, which is a fundamental condition for process feasibility. An “accuracy life” criterion is introduced and justified to quantitatively evaluate the safety margin of the technological system and to limit cutting parameters. ***Scientific novelty.*** For the first time, the existence of an absolute setting coordinate has been analytically proven based on a system of axioms formalizing the non-stationarity of the tolerance zone and the rigidity of a *CNC* system. It is shown that the “0–0 boundary” is the only point that guarantees process determinism under parametric uncertainty. ***Practical significance.*** The developed theory provides a methodological basis for creating digital twins of operations and intelligent *CAM* modules that automatically calculate setting parameters. This eliminates trial machining cycles, reduces the time required for the production process, and paves the way for the implementation of untended technologies in small- and medium-scale production.

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Introduction

The current stage of mechanical engineering development is characterized by the rapid digitalization of design procedures. At the pre-production engineering (*PPE*) stage, *CAD*/*CAM* systems have reached impressive levels of advancement: 3D modeling, automated process design, toolpath calculation, and *CNC* machine tool code generation have become commonplace. The *PPE* design process is completely digitalized. The engineer works with a virtual model of a part, workpiece, and toolpath.

*** Corresponding author**
Akintseva Alexandra V., Ph.D. (Engineering), Associate Professor
South Ural State University (National Research University),
76 Lenin Ave.,
454080, Chelyabinsk, Russian Federation
Tel.: +7 922 633-56-57, **e-mail:** akintsevaav@susu.ru

However, the transfer of this virtual model to a real workshop floor raises a fundamental challenge. The virtual toolpath calculated by the *CAD*/*CAM* system does not match the actual interaction between the tool and the workpiece, as it fails to take into account the most important factor – the interaction of the tool with the unstable physical reality: allowance fluctuations in a batch of workpieces, positioning errors, elastic deformations of the process system under the action of cutting forces, and dimensional tool wear. *CNC* systems receive a command to move the machine unit to the calculated point, but the exact location of the tool tip and the machined surface, considering all the above factors, remains unknown until the first part is measured [1, 2].

A "digital discontinuity" occurs at the intersection between the ideal digital design and the actual physical process. Its essence is as follows:

– on one side of the discontinuity (what has already been digitized): the part geometry, the toolpath, the control program (*G*-code), and the machine kinematics;

– on the other side of the discontinuity (what cannot be digitally modeled yet): actual cutting forces and elastic deformations they cause, the actual position of the workpiece tolerance zone in the machine coordinate system (*MCS*), which varies from part to part, positioning errors, tool wear, and the geometric inaccuracy of the machine.

The key link between the two sides is the dimensional setting of the machine, i.e., determining the absolute coordinate the tool tip should reach to guarantee the tolerance for the entire batch of parts. Currently, this link remains a "blind spot" in the digital chain, as there are no tools available for calculating it. Process engineers must rely on empirical methods: assigning coordinates "by eye", making a test piece, measuring it, and entering a correction, which both increases the time and cost of production planning and is fundamentally incompatible with the concept of unmanned technologies.

Unmanned technologies currently involve the creation of a "smart enterprise", where every technical decision is made based on a digital model, and processing parameters are calculated automatically, before the machine is started. This concept cannot be implemented without a complete digital twin of the operation, taking into account unstable factors. Digital twins are based on the mathematical theory that establishes a deterministic relationship between the input parameters and desired tool coordinates. The absence of such a theory creates the very "digital discontinuity".

To understand the causes of this "digital discontinuity", let us examine existing approaches to accuracy prediction and dimensional setting. The automation of *CNC* machining accuracy control is discussed in [1]. The current scientific paradigm can be represented by three main research areas.

The first area covers modeling of individual errors: geometric errors of machine tools [2–4], dimensional wear of tools [5, 6], and elastic deformations [7, 8]. The works in this area [2–4] consider a geometric algorithm for cutting modeling [2], review methods for compensating for geometric errors [3], and predict errors in milling thin-walled parts [4]. Wear studies are presented by a mathematical model of part size changes during batch processing [5] and a tool condition monitoring system based on the *Kalman* filter [6]. The influence of stress-strain states on the accuracy of non-rigid parts is studied in [7], and the error in setup schemes – in [8].

However, these studies are fragmentary and do not provide a comprehensive approach. Foreign studies are also limited in scope: compensation for thermal deformations [9] does not take into account elastic deformations; a review of compensation methods [3] requires expensive equipment; compensation for errors in milling thin-walled parts [4] is unsuitable for serial production. In [10], the authors note the lack of comprehensive models integrating all types of errors. More recent reviews [11] confirm the need to develop virtual systems for machining operations.

Classical works on worst-case tolerance analysis laid the foundation for a deterministic approach to dimensional chains [12–14]. However, these methods operate with stationary dimensional chains, while the proposed theory is the first to extend the worst-case principle to a non-stationary tolerance zone in the absolute coordinate system of the machine tool. Recent studies on tolerance analysis [15] also confirm the relevance of developing deterministic methods.

The concept of digital twins in manufacturing has been actively developed in recent years [16–19]. Works [16–17] and [18] laid the theoretical foundations, while [19] adapted the design and manufacturing concept. However, current digital twins are mainly focused on geometric and thermal modeling, omitting the deterministic calculation of tool setting coordinates. Only recent studies [20] are beginning to integrate digital twins with the tasks for monitoring tool conditions in *CNC* machines, confirming the relevance of the proposed approach.

The second area covers active and adaptive control methods: active monitoring [21–22], modelling of the process system (*PS*) accuracy [23], and cutting control [24–28]. An intelligent cutter with active temperature and size monitoring devices is presented in [21]. A set of control and measurement procedures for automated manufacturing on *CNC* machines, allowing for automatic wear correction, is developed in [22]. Modelling of *PS* accuracy indicators is considered in [23]. A model for calculating the cutting depth in an automatic feed cycle is proposed in [24]. However, such systems require expensive equipment and are reactive in nature. Active vibration compensation [25] requires machine tool modernisation; intelligent monitoring systems [26] have not left the laboratory; indirect measurement of cutting force [27] is not accurate enough; while adaptive systems [28] are effective only for rough machining.

The third area covers probabilistic simulation modelling [29]. Statistical methods provide only a probabilistic estimate, rather than a deterministic solution. These include research on dimensional accuracy analysis, such as a methodology for automated process accuracy assurance [30], which formalises the verification of the dimensional structure at the pre-production engineering stage. However, such methods do not take into account the probabilistic nature of errors and system rigidity, which limits their use for the guaranteed calculation of setting coordinates. The estimation of allowance variations using the *Monte Carlo* method [31] does not guarantee accuracy; while probabilistic models [32] are difficult to interpret.

The analysis of the scientific and technical literature reveals that, despite the research on accuracy prediction and automation of the dimensional settings of *CNC* machines, there is still no comprehensive theory linking all factors together. The conducted analysis allows drawing the following conclusions:

The analysis of the scientific and technical literature reveals that, despite the research on accuracy prediction and automation of the dimensional settings of *CNC* machines, there is still no comprehensive theory linking all factors together. The conducted analysis allows drawing the following conclusions:

1. Fragmented nature. Studies cover either individual errors or control methods. There is still no comprehensive theory linking all factors together.

2. Empirical nature of tool setting. Calculating tool coordinates is still an empirical procedure; existing approaches do not offer an analytical solution that ensures the machining of an entire batch from the first workpiece.

3. Failure to account for tolerance zone non-stationarity. Most models ignore the "floating" tolerance zone relative to the machine coordinate system, which is a crucial factor requiring deterministic consideration.

The ***objective of this study*** is to theoretically substantiate the existence and uniqueness of an absolute cutting tool coordinate that guarantees machining of an entire batch of parts without trial runs.

To accomplish this objective, the study addresses the following ***tasks***:

– to analyze the nature of "digital discontinuity" in pre-production engineering;

– to theoretically substantiate the existence and uniqueness of a stationary coordinate base ("0-0 boundary") as a condition for guaranteed contact between the tool and any workpiece;

– to develop an equation for balancing the tolerance zone and the sum of multi-component errors, which determines the fundamental feasibility of the process;

– to introduce the "accuracy resource" criterion for quantitatively assessing the robustness margin of a process system and limiting cutting modes;

– to develop a closed system of equations for calculating the absolute tool coordinate, which creates a methodological basis for constructing digital twins and transitioning to unmanned technologies.

Methods

The theoretical justification is based on a systems approach and an axiomatic method, which allows for the rigorous formalisation of initial assumptions. To develop the theory, let us adopt the following assumptions derived from mechanical engineering practice.

Axiom 1 (of the batch of parts). A batch of parts is a set of workpieces with variable dimensions of process bases and allowances, but their deviations lie within known limits defined by drawings and previous operations.

Axiom 2 (of the machine coordinate system). The coordinate system of a *CNC* machine is constant (stationary) for all workpieces being machined in the batch.

Axiom 3 (of the non-stationarity of the tolerance zone). The position of the tolerance zone for a given operational dimension relative to the machine coordinate system changes with the placement of each new workpiece. This variation is caused by errors in the workpiece shape and setup, as well as fixture errors and machine geometric inaccuracies.

In classical mechanical engineering theory, the term «base error» refers to deviations associated with datum misalignment. However, in *CNC* machining, during dimensional settings for an entire batch of parts, a wider range of factors should be taken into account, including workpiece shape errors within the batch, clamping deformations, fixture setup errors, and machine geometric inaccuracies. All these factors combined contribute to a shift (floating) in the tolerance zone of operational dimensions in the absolute coordinate system of the machine.

To describe this combined effect, the proposed theory introduces a broader concept of **workpiece positioning error** (Δ_{pos}). This term encompasses all factors that cause the non-stationarity of the tolerance zone for operational dimensions in the coordinate system of the *CNC* machine when changing workpieces in a batch. The magnitude of this error is limited by a maximum value $\mathbf{\Delta max}$ ***pos*** for a given batch.

Classical mechanical engineering technology (see, for example, the works of *A.A. Matalin*, *B.S. Balakshin*, *V.S. Korsakov*) uses the concepts of "base error" (ε_b) and "setup error" $\varepsilon_{set} = \varepsilon_b + \varepsilon_{clamp} + \varepsilon_{fix}$ to describe the deviations that arise during workpiece setup, where ε_{clamp} represents the clamping error and ε_{fix} represents the fixture error. However, this apparatus is insufficient for addressing problems of the dimensional setting of *CNC* machines and implementing unmanned technologies, since the following factors should be additionally taken into account: the error in the shape of base workpiece surfaces within a batch (ε_{bss}); the geometric inaccuracy of the machine and its thermal deformations (ε_{mach}); the positioning error of *CNC* actuators (***Ttp***); the wear of supporting elements in the fixture (ε_{wear}).

Therefore, a broader term, **workpiece positioning error** (Δ_{pos}), has been introduced to characterise the total shift in the tolerance zone of operational dimensions in the absolute coordinate system of the machine when changing workpieces in a batch. Table 1 shows the relationship between classical and proprietary terms. Thus, the proposed concept of Δ_{pos} does not conflict with the classical terminology, but rather expands it to include *CNC* machining conditions where all factors causing the non-stationarity of the tolerance zone should be taken into account in the absolute coordinate system.

Axiom 4 (of geometry generation). Surfaces are machined when the cutting tool tip comes into contact with the workpiece. The actual position of the machined surface differs from the programmed position of the tool tip by the amount of elastic deformations in the process system y_d. For simplicity of analysis, all errors affecting machining accuracy are classified into three groups based on their impact on the stationarity of the workpiece and cutting tool tip positions.

Positioning errors (Δ_{pos}). These factors cause **"**floating**"** (non-stationarity) of the tolerance zone of the workpiece dimension relative to the machine bench. These include: fixture setup error, error in the shape of workpiece base surfaces, clamping error, and other factors affecting the positioning of the tolerance zone of the operational dimensions in the *CNC* coordinate system.

Tool errors (Δ_{tool}). These factors cause **"**floating**"** of the cutting tool tip relative to its holder (tool zero). They include: tool overhang tolerance and tool wear.

Errors caused by elastic deformations (y_d). These errors occur directly during the cutting process due to the finite rigidity of the process system under the influence of the cutting force component in the direction of the maintained dimension. They affect the position of the cutting tool tip within the tolerance zone.

The proposed axiomatics is appropriate if the following conditions are met: elastic deformations are linear (the load is no more than 0.5 times of the nominal value), the thermal drift of the machine does not exceed 0.02 mm (continuous operation time is ≤1 hour or thermal compensation is present), the vibration

Table 1

Correspondence of terms characterizing the deviation of the workpiece position in the machine coordinate system

Classic term (*Matalin*, *Balakshin*, *Korsakov*)	Error components	Authors' term
Base error (ε_{loc})	Mismatch between measurement and process bases	Included in $\boldsymbol{\Delta}_{pos}$
Clamping error ($\boldsymbol{\varepsilon}_{clamp}$)	Deformations of the workpiece and fixture elements under the action of clamping forces	Included in $\boldsymbol{\Delta}_{pos}$
Fixture error ($\boldsymbol{\varepsilon}_{fix}$)	Fixture manufacturing and setup accuracy	Included in $\boldsymbol{\Delta}_{pos}$
Setup error ($\boldsymbol{\varepsilon}_{set} = \boldsymbol{\varepsilon}_b + \boldsymbol{\varepsilon}_{clamp} + \boldsymbol{\varepsilon}_{fix}$)	Total deviation associated with basing and clamping	Is part of $\boldsymbol{\Delta}_{pos}$
– (not included in $\boldsymbol{\varepsilon}_{set}$)	Error in the shape of the workpiece base surface ($\boldsymbol{\varepsilon}_{bss}$)	Included in $\boldsymbol{\Delta}_{pos}$
– (not included in $\boldsymbol{\varepsilon}_{set}$)	Wear of the fixture support elements ($\boldsymbol{\varepsilon}_{wear}$)	Included in $\boldsymbol{\Delta}_{pos}$
– (not included in $\boldsymbol{\varepsilon}_{set}$)	Geometric inaccuracy of the machine and thermal deformations ($\boldsymbol{\varepsilon}_{mach}$)	Included in $\boldsymbol{\Delta}_{pos}$
– (not included in $\boldsymbol{\varepsilon}_{set}$)	Positioning error of *CNC* actuators (*Ttp*)	Included in $\boldsymbol{\Delta}_{pos}$ and $\boldsymbol{\Delta}_{tool}$

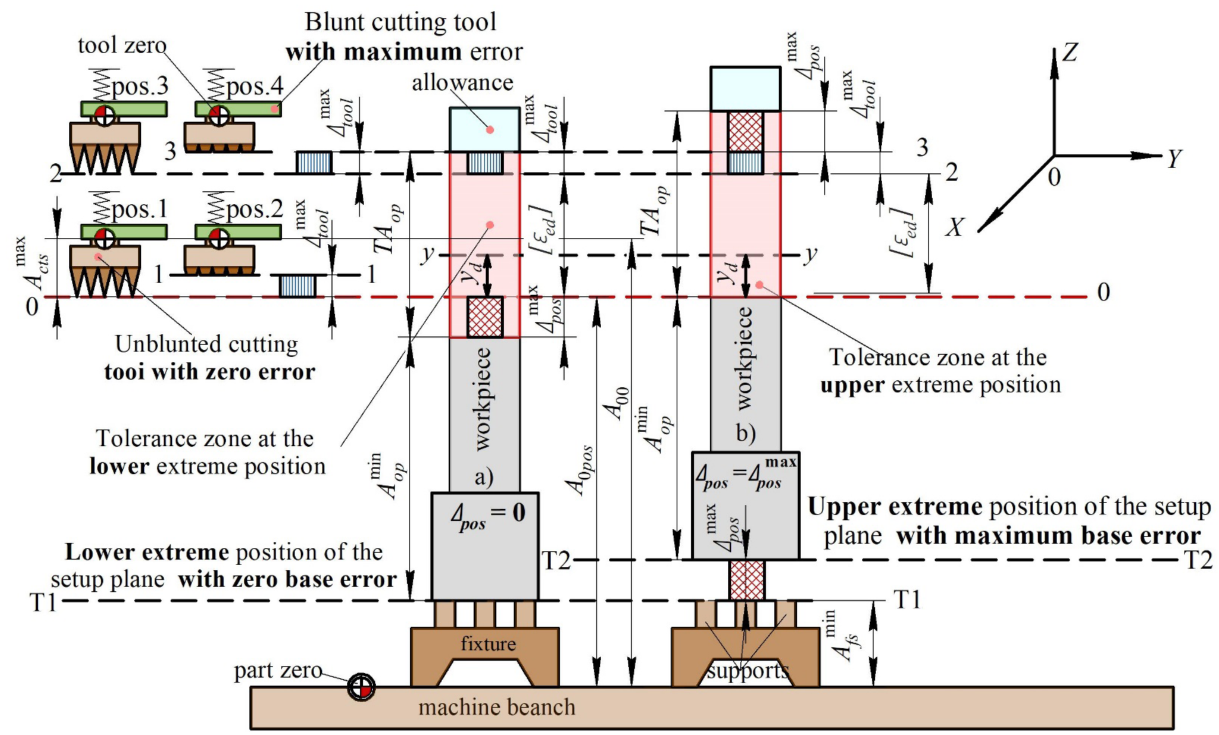
amplitude is less than 0.1 $\boldsymbol{TA}_{op}$, and the workpiece material has normal thermal conductivity (steel, cast iron). If these conditions are not met (titanium alloys, thin-walled parts, long-term machining), the model should be supplemented with thermal and dynamic calculation blocks.

This classification forms the basis for a calculation scheme (see below) designed to analytically determine the absolute coordinate of the cutting tool tip during its final pass. The scheme was developed using the example of planar machining during milling or surface grinding operations and reflects the relative positions of the workpiece, tool, and tolerance zone in the *CNC* machine coordinate system.

The workpiece is located in a fixture on three supports forming the setup plane (see Fig.). The fixture is located on the machine bench plane, which is taken as the workpiece zero, since the bench position remains constant during the machining of an entire batch of parts. However, when changing between workpieces, the position of the setup plane in the *CNC* coordinate system is unstable due to various errors affecting the workpiece positioning and the actual position of the tolerance zone of the operational dimension along the *Z*-axis.

Considering that the tolerance zone fluctuations in a batch of parts occur within a limited range, the calculation scheme shows the positioning of a workpiece with a tolerance zone in two extreme positions along the *Z*-axis: in the lower (*a*) and upper (*b*) extreme positions. In the lower extreme position (*a*), the workpiece has zero total positioning error, i.e. $\boldsymbol{\Delta}_{pos} = 0$. In the upper extreme position (*b*), the positioning error is maximum along the *Z*-axis: $\left(\Delta_{pos} = \Delta_{pos}^{\max}\right)$. In the lower extreme position of the workpiece (*a*), the setup plane passes through the fixture supports and is designated by the **T1-T1** boundary. The distance from the workpiece zero (bench plane) to the **T1-T1** boundary is equal to the minimum distance $A_{fs}^{\min}$ from the fixture base to the supports of the setup plane.

In the upper extreme position of the workpiece (*b*), the setup plane passes along the **T2-T2** boundary. The distance from the **T1-T1** boundary to the **T2-T2** boundary is equal to the maximum positioning error. The maintained (operational) dimension is designated $\boldsymbol{A}_{op}$. In the diagram, it is shown as the minimum dimension with tolerance $\boldsymbol{TA}_{op}$. The operational dimension $\boldsymbol{A}_{op}$ is associated with the process base – the setup plane of the workpiece.



Calculation scheme for determining the absolute coordinate of the tool tip Δ_{0pos} at the extreme positions of the workpiece:

a – lower position ($\Delta_{pos} = 0$); *b* – upper position $\left(\Delta_{pos} = \Delta^{max}_{pos}\right)$

The tolerance zone of the operational dimension A_{op} becomes **non-stationary** relative to the workpiece zero point with each new workpiece from the batch being machined. This means that when changing workpieces, the same tolerance zone boundary (for example, its lower boundary) occupies different positions along the *Z*-axis relative to the machine bench.

The non-stationarity of the tolerance zone position is determined by the positioning error Δ_{pos}, which includes: the error associated with the tolerance TA_{fs} of the fixture size A_{fs}; the error in the shape of the workpiece base surface ε_{bss}; the error in the clamping ε_{clamp} of the workpiece in the fixture; the error ε_{mgi} caused by the geometric inaccuracy of the machine.

To calculate the absolute coordinate of the cutting tool tip under conditions of workpiece position fluctuation, let us consider the maximum permissible position of a non-blunted cutting tool with maximum overhang ***Amax cts***. Obviously, the tool tip on the final pass should not fall below the lower limit of the tolerance zone. The most dangerous situation occurs when the workpiece is in its upper extreme position (*b*). In this case, the lower limit of the tolerance zone is located at the maximum distance from the workpiece zero point along the *Z*-axis.

To limit the lower position of the tool tip, let us draw (*b*) a **0-0 boundary** through the lower limit of the tolerance zone at the upper position of the workpiece. After stock removal, on the final pass, the programmed movement of the tool tip should end at the *0-0* boundary. In this case, the tool tip will not be able to cross this boundary, which guarantees compliance with the lower limit of the tolerance zone.

As can be seen from (*a*), the 0-0 boundary cuts off a part of the tolerance zone, forming an inaccessible zone for the tool tip **of size Δmax *pos*** in the tolerance zone of the operational dimension at the lower position of the workpiece, which also guarantees safety at the lower limit.

Let us align (position **1** in Fig.) the tip of a sharp cutting tool with maximum overhang with the 0-0 boundary. The distance A_{0pos} between the workpiece zero point and the 0-0 boundary is the desired absolute coordinate of the cutting tool tip and can be calculated using the formula:

$$A_{0\,pos} = A^{min}_{pos} + \Delta^{max}_{op} + A^{min}, \qquad (1)$$

where $A_{pn}^{\min}$ is the minimum distance from the machine table (workpiece zero) to the fixture setup plane (**T1-T1** boundary); $\Delta_{pos}^{\max}$ is the maximum workpiece positioning error; $A_{op}^{\min}$ is the minimum operational dimension (the distance from the setup plane (process base) to the lower limit of the tolerance zone of the machined dimension).

When machining with a blunted tool (with minimum overhang and maximum tool error $\left(\Delta_{tool}^{\max}\right)$, the tool tip (position **2** in Fig.) coincides with the 1-1 boundary, which is located safely above the 0-0 boundary by $\left(\Delta_{tool}^{\max}\right)$, which is also safe for the lower limit of the tolerance zone.

Let us consider ensuring accuracy with respect to the upper limit of the tolerance zone. Under the action of elastic deformations $\boldsymbol{y_d}$ in the process system, the tool tip shifts upward along the *Z*-axis. With excessive elastic deformation, the tool tip can cross the upper limit of the tolerance zone, which would lead to rejects. The most dangerous situation occurs when the upper limit of the tolerance zone together with the workpiece are in the lowest position (*a*).

Let us draw a maximum permissible boundary 3-3 of the tool tip position through the upper limit at the lower position of the workpiece. Let us align (position **4**) the tip of the blunted tool with the 3-3 boundary. In this case, the tip of the sharp tool (position **3**) will be inside the tolerance zone and will coincide with the 2-2 boundary at a safe distance from the upper limit. The distance between the 2-2 and 3-3 boundaries is equal to the maximum tool error $\left(\Delta_{tool}^{\max}\right)$.

The distance between the 0-0 and 2-2 boundaries is equal to the maximum permissible value of elastic deformation $[\boldsymbol{\varepsilon}_{ed}]$ on the final pass of the tool. The value of $[\boldsymbol{\varepsilon}_{ed}]$ is controlled by the cutting parameters by limiting the cutting force, followed by calculation of the permissible value of elastic deformation $\boldsymbol{y_d}$ for a given rigidity of the process system.

From the dimensional diagram (*a*) it can be seen that the tolerance TA_{op} of the operational dimension at the lower position of the workpiece is equal to the sum of three errors:

$$TA_{op} = \Delta_{pos}^{\max} + \Delta_{tool}^{\max} + [\varepsilon_{ed}].$$

The physical meaning of this equality is that the operational tolerance compensates for the three-component total error.

At the upper position of the workpiece (*b*), according to the Figure, the 2-2 and 3-3 boundaries cut off the same error components from the tolerance zone, and the operational tolerance TA_{op} is also equal to the sum of these three errors.

Let us introduce a key concept of the proposed theory.

Definition 1. The 0-0 boundary is an imaginary plane (line) in the coordinate system of a *CNC* machine, the position of which remains unchanged (stationary) during the machining of an entire batch of parts. It serves as a single coordinate base to which both the workpiece position (through its process base) and the tool position (through its overhang) are linked.

Theorem 1 (On the existence of a stationary 0-0 boundary).

A stationary boundary can always be defined in the machine coordinate system for any batch of workpieces with the known maximum possible positioning error **Δmax pos**, such that if the tip of the cutting tool reaches this boundary during the finishing pass, the entire batch will be machined without defects at the lower limit (there will be no «undercuts»).

Proof. Let us consider how the lower boundary of the tolerance zone can be positioned for different workpieces in the batch. Due to positioning errors, it occupies a new position each time. Let us denote the set of these positions as $\boldsymbol{H_i}$. According to **Axiom 3**, this set is not infinite, but rather bounded above by a certain highest position – $\boldsymbol{H_{\max}}$. This is the worst-case scenario, where the sum of all errors has raised the lower tolerance limit to its highest point (closest to the tool).

Let us select a plane in the machine coordinate system that passes through this point $\boldsymbol{H}_{\mathbf{max}}$. For each workpiece in the batch, its own lower limit, $\boldsymbol{H}_i$, will be on or below this plane. If we program the tool so that its tip is on this plane at the end of machining, for the most "inconvenient" workpiece (with $\boldsymbol{H}_i = \boldsymbol{H}_{\mathbf{max}}$), the tool will just touch the tolerance limit. However, for all other workpieces, it is guaranteed to enter the workpiece body and remove the stock allowance. Therefore, this plane (the 0-0 boundary) exists.

Theorem 2 (On the uniqueness of the 0-0 boundary).

The position of the 0-0 boundary, which is drawn through the highest possible position of the lower boundary of the tolerance zone $\boldsymbol{H}_{\mathbf{max}}$, is the only one that ensures the absence of defects with the minimum required metal removal.

Proof. Suppose we draw the $\boldsymbol{G}$ boundary higher than $\boldsymbol{H}_{\mathbf{max}}$ (closer to the upper tolerance limit, i.e., with a larger coordinate along the *Z*-axis). In this case, the tool will cut deeper than necessary for the workpiece with the worst positioning ($\boldsymbol{H}_i = \boldsymbol{H}_{\mathbf{max}}$). This increases the load on the system and the risk of exceeding the upper tolerance limit due to elastic deformations, but may be formally acceptable.

Suppose we draw the $\boldsymbol{G}$ boundary lower than $\boldsymbol{H}_{\mathbf{max}}$ (further away from the tool, i.e., with a smaller coordinate along the Z-axis). In this case, the tool tip at the end point $\boldsymbol{G}$ will be higher than its lower tolerance limit $\boldsymbol{H}_{\mathbf{max}}$ for the worst-case workpiece position ($\boldsymbol{H}_i = \boldsymbol{H}_{\mathbf{max}}$). This means the tool simply will fail to touch the workpiece – no stock allowance will be removed, and the part will be scrapped due to undercutting. Therefore, the only position that ensures tool contact with any workpiece in the batch is the $\boldsymbol{H}_{\mathbf{max}}$ parameter.

Consequence 1. The 0-0 boundary serves as **a stationary coordinate base** in the *CNC* system. The dimension from the workpiece zero (machine bench) to this boundary ($\boldsymbol{A}_{0pos}$) and the dimension from the tool zero to this boundary ($\boldsymbol{A}_{cts}$) are the desired dimensional setting parameters, which can be calculated analytically (formula (1)).

The presence of the 0-0 boundary solves the problem of the lower tolerance limit, but it leaves open the question of the upper limit. Elastic deformations can "push" the machined surface upward, beyond the tolerance limits.

Definition 2. The accuracy resource [$\boldsymbol{\varepsilon}_{ed}$] is the portion of the tolerance zone of an operational dimension that remains after subtracting the maximum possible static errors (positioning and tooling) from it. This is the "reserve" that can be spent on elastic deformations and other dynamic effects without exceeding the upper tolerance limit.

Theorem 3 (Balance equation).

A batch of parts can be machined without defects only if the accuracy resource [$\boldsymbol{\varepsilon}_{ed}$] is non-negative and the actual elastic deformation does not exceed its value.

Proof. The operational dimension $\boldsymbol{A}_{op}$ is the distance between the design base and the machined surface. In the worst-case scenario, when all errors accumulate, the dimension can reach its maximum value: the maximum operational dimension equals the sum of the position of the process base at the worst possible positioning and the value of the removed layer, taking into account elastic forces in the process system.

The condition of not exceeding the upper tolerance limit is described by the following inequality:

$$TA_{op} \geq \Delta_{pos}^{\max} + \Delta_{tool}^{\max} + y_d \, . \tag{2}$$

If we move the static components to the left side, we obtain:

$$[\varepsilon_{ed}] = TA_{op} - \left(\Delta_{pos}^{\max} + \Delta_{tool}^{\max}\right) \geq y_d \, . \tag{3}$$

The physical essence of the accuracy resource is dual:

Fundamental machinability indicator. If [$\boldsymbol{\varepsilon}_{ed}$] < 0, the sum of the static errors alone (positioning and tooling) already exceeds the tolerance. This means that the process is not feasible with the given positioning scheme, and changing the cutting conditions cannot compensate for this discrepancy. The technological process should be revised.

Cutting mode limiter. Since elastic deformation depends on the cutting force, the condition $[\boldsymbol{\varepsilon}_{ed}] \geq \boldsymbol{y}_d$ directly limits the permissible cutting force, and therefore the feed and depth during the finishing pass. This enables the calculation of cutting conditions that ensure accuracy.

Based on the proven theorems and the analysis of dimensional relationships, we obtained a closed system of equations representing a mathematical model for calculating dimensional setting parameters:

1. Overall positioning error:

$$\Delta_{pos}^{\max} = TA_{fs} + \varepsilon_{bss} + \varepsilon_{clamp} + \varepsilon_{wear} + Ttp + \varepsilon_{mach}, \tag{4}$$

where $\boldsymbol{TA}_{fs}$ is the dimensional tolerance characterizing the manufacturing accuracy and the setup error of the fixture itself on the machine bench; $\boldsymbol{\varepsilon}_{bss}$ is the error in the shape of the workpiece base surfaces obtained in the previous operation; $\boldsymbol{\varepsilon}_{clamp}$ is the clamping error caused by elastic deformations of the workpiece and fixture elements under the action of clamping forces; $\boldsymbol{\varepsilon}_{wear}$ is the error associated with the wear of the fixture support elements during operation; $\boldsymbol{Ttp}$ is the positioning error of the *CNC* actuators; $\boldsymbol{\varepsilon}_{mach}$ is the generalized error associated with the geometric inaccuracy of the machine, its wear, and thermal deformations that affect the relative position of the components.

2. Overall tool error:

$$\Delta_{tool}^{\max} = TA_{cts} + \varepsilon_{ctw} + Ttp, \tag{5}$$

where $\boldsymbol{TA}_{cts}$ is the cutting tool setup (overhang) tolerance specified by the symmetric tolerance ($\boldsymbol{A}_{cts} \pm \pm 0.5TA_{cts}$); $\boldsymbol{\varepsilon}_{ctw}$ is the error caused by the dimensional wear of the cutting tool over its service life; $\boldsymbol{Ttp}$ is the *CNC* positioning error (program feed movement tolerance) characterizing the accuracy of the machine when executing a specified movement and typically specified by a symmetrical tolerance ($tp \pm 0.5Ttp$).

The same *CNC* positioning error value is taken into account both in positioning errors (formula (4) and in tool errors (formula (5)), since it affects both components of dimensional setting).

3. The coordinate of the 0-0 boundary from the machine bench is determined by formula (1).

4. Absolute coordinate of the tool zero (from the machine bench):

$$A_{00} = A_{0\,pos} + A_{cts}^{\max}, \tag{6}$$

where $\boldsymbol{A}_{0pos}$ is the coordinate of the 0-0 boundary from the machine bench (the distance from the machine bench to the stationary 0-0 boundary); $A_{cts}^{\max}$ is the maximum tool overhang (the maximum possible distance from the end of the holder (tool zero) to the tip of the cutting edge).

5. Balance equation (feasibility condition) – formula (2).

6. Accuracy resource – formula (3).

7. Limiting cutting modes by accuracy:

$$y_d \leq [\boldsymbol{\varepsilon}_{ed}]. \tag{7}$$

All the parameters in equations (1–7) are known or can be determined at the PPE stage. Cutting forces and rigidity can be determined empirically using reference data or calculated using known methods, which allows determining the desired coordinate $\boldsymbol{A}_{0pos}$ and the maximum allowable load during the finishing pass before starting the machine.

Several conditions should be met to apply the proven theorems on the existence and uniqueness of the absolute tool coordinate (0-0 boundary) in practice. These conditions create a digital environment where calculating the absolute coordinate becomes possible and deterministic. A key requirement is the availability of both mathematical models and digital passports for all elements of the process system, serving as sources of reliable initial data.

These conditions should be ensured at the *PPE* stage and are input parameters for the proposed theory. Each condition is supported by a relevant digital passport.

1. Availability of a digital model of positioning errors

The calculation of $\Delta_{pos}^{\max}$ using equation (4) provides for a mathematical description of all components. The following data sources are used:

– **the digital passport for the fixture** contains information about $\boldsymbol{TA_{fs}}$ (fixture manufacturing accuracy), $\boldsymbol{\varepsilon_{clamp}}$ (certified or calculated clamping deformations), and data on the rigidity of fixture elements that affect clamping deformations;

– **the digital passport for the workpiece (batch of parts)** contains statistical data on the error in the shape of base surfaces ($\boldsymbol{\varepsilon_{bss}}$), the accuracy of the previous operation ($\boldsymbol{\varepsilon_{wear}}$), and the distribution law for allowances in the batch, enabling one to determine $\Delta_{pos}^{\max}$ and the position $\boldsymbol{H_{\max}}$;

– **the digital passport for the machine** contains accuracy characteristics ($\boldsymbol{\varepsilon_{mach}}$), including geometric accuracy and positioning error, as well as data on the rigidity of nodes, which are necessary for modelling elastic deformations.

2. Availability of a digital model of tool errors

To calculate $\Delta_{tool}^{\max}$ using equation (5), the following data are needed:

– **the digital passport for the cutting tool** contains data on the tool overhang tolerance ($\boldsymbol{TA_{cts}}$), the expected dimensional wear ($\boldsymbol{\varepsilon_{ctw}}$) as a function of the cutting path or machining time, and the geometric parameters affecting the cutting forces.

– **the digital passport for the tool setting (presetter)** contains data on the accuracy of the tool presetting outside the machine (***Ttp***).

3. Availability of a digital model of elastic deformations

Balance equation (2) and constraint (3) suggest that we should know the value of elastic deformations y_d. The following data sources are used:

– **the digital passport for the machine** contains data on the rigidity of the process system (compliance matrix in the work area);

– **the digital passport for the workpiece material** contains data on the mechanical properties (hardness, tensile strength) necessary to calculate the cutting force using empirical relationships or other proprietary **methods**;

– **the digital passport for the tool** contains coefficients in the cutting force equations, which depend on the tool geometry and cutting material.

4. Availability of a digital metrological infrastructure

Digital passports for measuring instruments, containing data on accuracy, measurement range, and verification frequency, are necessary for model verification and, potentially, system self-learning in the concept of cyber-physical systems. Measurement results (from trial or first batch parts) should be digitally entered into the system for comparison with forecasts and, if necessary, model adjustment (*Bayesian* updating).

5. Stationarity of the machine coordinate system (*MCS*)

This condition enshrined in **Axiom 2** implies that all calculations are tied to a constant physical coordinate system of the machine (bench or spindle). Any machine deformations should either be compensated for or kept within negligible limits. Deformation data can be recorded in the digital passport for the machine as a function of operating time and temperature.

Sufficient conditions (feasibility criteria): if the necessary conditions are met (digital models are created and parameterized), the existence of an absolute coordinate is guaranteed if the following sufficient conditions, derived from the proven theorems, are met:

– **condition of the non-negativity of the accuracy resource ($[\boldsymbol{\varepsilon_{ed}}] \geq 0$).** According to balance equation (2) and Theorem 3, if $\boldsymbol{TA_{op}} \geq \Delta_{pos}^{\max} + \Delta_{tool}^{\max}$, the accuracy resource exists. Otherwise ($[\boldsymbol{\varepsilon_{ed}}] < 0$), the sum of static errors exceeds the tolerance, and the machining process is fundamentally unfeasible. The 0-0 boundary calculated according to Theorem 1 loses practical significance in this case, since the upper tolerance limit will be exceeded even before taking elastic deformations into account;

– **error balance condition** ($y_d \leq [\varepsilon_{ed}]$). Even with a positive accuracy resource, the absolute coordinate calculated according to equation (1) guarantees the production of a suitable part only if the actual elastic deformations y_d (depending on the cutting conditions) do not exceed this resource. This condition connects the static setup calculation with the dynamics of the cutting process;

– **contact determinacy condition (H_{max} principle).** According to Theorems 1 and 2, the uniqueness and existence of the 0-0 boundary are guaranteed only if it passes through the highest point of the lower tolerance limit $H_{\max}$. This condition eliminates setup variability and makes the result prediction deterministic for the entire batch.

The proposed theory establishes a rigid logical chain in which digital passports serve as a fundamental source of reliable data: digital passports (for a fixture, tool, machine, workpiece, or measuring instrument) → parameterization of digital error models → calculation of $\Delta_{pos}^{\max}$, $\Delta_{tool}^{\max}$ and $[\varepsilon_{ed}]$ → verification of sufficient conditions ($[\varepsilon_{ed}] \geq 0$) → application of Theorems 1 and 2 to calculate a unique absolute coordinate A_{0pos} → limiting cutting modes by the condition $y_d \leq [\varepsilon_{ed}]$ → manufacturing the part → accuracy confirmation (verification of the model by measurement data) → refinement of digital passports and models (optional, for self-learning systems).

Any violation of these conditions (absence of a digital passport for any system element, incomplete data therein, or a negative accuracy resource) breaks this chain and returns the process to the realm of empirical uncertainty, thereby reproducing the "digital discontinuity" this study aims to overcome.

Numerical example: calculating the absolute coordinate of a tool for milling the plane of a case-shaped part.

To demonstrate the operability of the proposed theoretical apparatus and the application of formulas (1)–(7), let us perform an end-to-end calculation for the **milling process of the plane of a case-shaped part** on a *CNC* machine. The machining is performed using a face mill, with the maintained dimension being the distance between the setup plane (process base) and the machined surface.

Stage 1. Calculating the components of overall errors

1.1. We calculate the positioning error using formula 4, assuming the following parameters for our case: TA_{fs} = 0.03 mm (fixture manufacturing accuracy); ε_{bss} = 0.02 mm (error in the shape of the workpiece base surface); ε_{clamp} = 0.01 mm (deformations during clamping); ε_{wear} = 0.005 mm (wear of the fixture supports); T_{tp} = 0.005 mm (*CNC* positioning error); ε_{mach} = 0.02 mm (geometric inaccuracy of the machine).

$$\Delta_{pos}^{\max} = 0.03 + 0.02 + 0.01 + 0.005 + 0.005 + 0.02 = 0.07 \text{ mm.}$$

1.2. We calculate the tool error using formula 5 and assume the following values for this example: TA_{cts} = **0.02 mm** (tool overhang tolerance); ε_{ctw} = **0.015 mm** (dimensional wear over the tool life); Ttp = **0.005 mm** (*CNC* positioning error).

$$\Delta_{tool}^{\max} = TA_{cts} + \varepsilon_{ctw} + Ttp = 0.02 + 0.015 + 0.005 = 0.04 \text{ mm.}$$

Stage 2. The process fundamental feasibility is verified using formula 3. We determine the accuracy resource:

$$[\varepsilon_{ed}] = TA_{op} - \left(\Delta_{pos}^{\max} + \Delta_{tool}^{\max}\right) = 0.012 - (0.07 + 0.04) = 0.01 \text{ mm.}$$

$[\varepsilon_{ed}] = 0.01 > 0$, therefore, **the process is fundamentally feasible**. A positive accuracy resource means that static errors do not exceed the tolerance, leaving a 0.01 mm margin for elastic deformations. If the accuracy resource turned out to be negative, this would indicate the impossibility of machining with the given positioning scheme and the selected tool – it would be necessary to revise the process (Table 2).

Stage 3. The «0-0 boundary» coordinate is calculated using formula 1:

$$A_{0\,pps} = A_{\min} + \Delta_{pos}^{\max} + A^{\min} = 80.0 + 0.07 + 120.0 = 200.07 \text{ . mm.}$$

Table 2

Initial data (realistic for small-batch production)

Parameter	Designation	Value	Note
Minimum operational dimension (drawing)	A_{op}^{min}	120.0 mm	distance from the base to the lower limit of the tolerance zone
Operational dimension tolerance	$\boldsymbol{TA_{op}}$	0.12 mm	tolerance zone as per IT9 for a dimension of 120mm
Maximum positioning error	Δ_{pos}^{max}	0.07 mm	calculated using formula (4)
Maximum tool error	$_{tool}^{max}$	0.04 mm	calculated using formula (5)
Minimum distance from the machine bench to fixture supports	$\boldsymbol{A_{min}}$	80.0 mm	from the fixture drawing
Maximum tool overhang	$\boldsymbol{l_{max}}$	50.0 mm	from the tool passport
Rigidity of the process system	$\boldsymbol{j}$	40 kN/mm	machine passport
Permissible elastic deformation	$[\boldsymbol{y_d}]$	to be defined	from condition (7)

Interpretation: The 0-0 boundary plane should be located at a distance of 200.07mm from the machine bench (workpiece zero). The cutter tip should be programmed to reach this coordinate during the finishing pass to ensure stock removal for any workpiece in the batch, including those with the worst positioning ($\boldsymbol{H_{max}}$).

Stage 4. The absolute coordinate of the tool zero is calculated using formula 6:

$$A_{0tool} = A_{pos} - l_{\max} = 200.07 - 50.0 = 150.07 \text{ mm.}$$

This is the distance from the machine bench to the spindle nose (tool holder), which should be specified during tool setup on the presetter or compensated for in the *CNC* system.

Stage 5. Limiting cutting modes by accuracy (formula 7)

According to condition (7), the permissible cutting force is:

$$P_y^{perm} \leq [y_d] \cdot j \,.$$

At $[\boldsymbol{y_d}] = [\boldsymbol{\varepsilon_{ed}}] = 0.01$ mm (we set the entire accuracy resource) and $\boldsymbol{j} = 40$ kN/mm:

$$P_y^{perm} \leq 0.01 \cdot 40 = 0.4 \text{ kN} = 400 \text{ N} \,.$$

According to the reference cutting force dependencies for face milling of grade 45 steel, this limitation corresponds, for example, to the following conditions: cutting depth $\boldsymbol{t} = 0.5$ mm, feed per tooth $\boldsymbol{s_z} = 0.05$ mm/tooth, cutting speed $\boldsymbol{v} = 150$ m/min. If these parameters are exceeded, y_d will be higher than 0.01mm and there will be the risk of exceeding the upper tolerance limit.

Stage 6. The balance is checked using formula 2:

$$0.12 \geq 0.07 + 0.04 + 0.01.$$

The equation is satisfied: 0.12 = 0.12, which means that with the set elastic deformation $\boldsymbol{y_d} = 0.01$mm (which is the maximum allowable one), the entire tolerance is fully utilized. To ensure quality, it is recommended to set $\boldsymbol{y_d}$ with a margin (e.g., 0.008 mm), which will result in $0.12 \geq 0.118$ and increase reliability.

To demonstrate the dependence of the result on the input parameters, let us consider three options for changing the initial data.

Option A (tightening the tolerance). If we reduce the operational dimension tolerance to TA_{op} = 0.08 mm (instead of 0.12), then $[\varepsilon_{ed}]$ = 0.08 – (0.07 + 0.04) = − 0.03. The process becomes **unfeasible** – formula (3) yields a negative accuracy resource. The process engineer should either modify the positioning scheme (decrease $\Delta_{pos}^{\max}$) or use a more precise tool (decrease $\Delta_{tool}^{\max}$).

Option B (increasing positioning error). If we increase $\Delta_{pos}^{\max}$ to 0.09mm (e.g., due to fixture wear), then $[\varepsilon_{ed}]$ = 0.12 – (0.09 + 0.04) = –0.01 mm. The process is unfeasible. **The fixture should be replaced or repaired.**

Option C (reducing system rigidity) Let us reduce the rigidity to j = 20 kN/mm (for example, when machining a non-rigid part). With the same tool life of $[\varepsilon_{ed}] = [\varepsilon_{ed}]$ = 0.01mm: P_y^{perm} = 0.01 · 20 = 200 N. The permissible cutting force is halved, requiring a significant reduction in feed or cutting depth. If the desired productivity is not achieved, the system rigidity should be increased (for example, by using guide bushes).

Conclusions from the numerical example.

1. All seven formulas (1)–(7) work in concert and form a closed calculation procedure: from error assessment → feasibility check → determining the 0-0 boundary and tool coordinates → limiting cutting modes → balance check.

2. The accuracy resource (formula 3) is a key diagnostic criterion: its negative value indicates the fundamental impossibility of machining even before starting the machine.

3. Sensitivity analysis shows how changes in tolerances, errors, or rigidity affect the final result, allowing process engineers to make grounded decisions at the *PPE* stage.

4. The calculation was fully completed during the pre-production engineering stage without trial runs. After setting the machine to A_{0pos} = 200.07 mm, the first workpiece in the batch should be suitable in size, confirming that the proposed theory is feasible for implementing unmanned technologies.

Results and discussion

The conducted theoretical study offers a new perspective on the dimensional setting of *CNC* machines. Unlike the common approach, which views tolerance instability as a source of uncertainty requiring probabilistic or adaptive methods, the proposed theory proves the existence of a deterministic solution. The key result is a mathematical proof that the machine coordinate system has a unique stationary plane – the 0-0 boundary – which guarantees tool contact with the workpiece regardless of the combination of positioning errors (**Theorems 1 and 2**).

The physical interpretation of this result suggests that the "worst-case" positioning (the highest position of the lower boundary of the zone $H_{\max}$) paradoxically introduces an element of determinism into the machining process. This limiting case becomes the reference point that transforms a non-stationary system into a stationary one relative to the machine coordinate system.

The proposed approach improves on the classical methods of *Worst-Case Tolerance Analysis* developed by *Greenwood* and *Chase* [12–13], as well as *Bjørke* [14]. Classical tolerance theory applies the «worst-case» principle to dimensional **variations** in a stationary coordinate system. The authors' contribution extends this principle to **a non-stationary tolerance zone**, which "floats" in the machine coordinate system from workpiece to workpiece. The "worst case" here is the highest position of the lower limit of the tolerance zone determined by positioning errors. This allows calculating the **absolute coordinate of the tool** in the machine space, rather than a dimensional tolerance. The introduced "accuracy resource" complements the classical approach by addressing the issue of the margin for elastic deformations and limiting cutting conditions.

Equally important is the balance equation (2), which serves as a criterion for the fundamental feasibility of the technological process. If the sum of static errors (positioning and tooling) exceeds the tolerance ($[\varepsilon_{ed}] < 0$), then, as follows from Theorem 3, the process cannot be implemented in principle, regardless of how perfect the cutting conditions are. This principle has important practical significance, as it allows for diagnosing process design errors at the earliest stage – even before setting machining conditions. In this sense, the proposed approach is more effective than empirical methods, which only allow diagnostics after the production and measurement of a trial batch.

The introduction of the “accuracy resource” criterion transforms the task of setting cutting conditions from a qualitative to a quantitative one. The $\boldsymbol{y_d} \leq [\boldsymbol{\varepsilon_{ed}}]$ condition not only limits the conditions but also provides process engineers with a specific “safety margin” that they can use. The larger the $[\boldsymbol{\varepsilon_{ed}}]$ value, the more resilient the process is to random fluctuations in material hardness or allowance, paving the way for optimizing productivity not based on empirical tables, but on a clear robustness criterion.

Notably, the proposed theory differs significantly from the existing approaches discussed in the literature review. Unlike fragmentary studies on individual error components [2–8], it offers a comprehensive view of the dimensional setting process. Unlike adaptive systems and active control methods [21–28], which only respond to deviations that have already occurred or require expensive equipment, the developed theory allows preventing defects at the design stage. Finally, unlike probabilistic methods [29, 31–32], which provide only a statistical assessment, the proposed theory offers a deterministic prediction of the result (suitable/rejected) for an entire batch of parts. Therefore, the theory fills the scientific and technical gap identified by many researchers [10, 26, 32].

The fundamental adaptation of the proposed theory for external turning operation requires addressing several challenging issues. Firstly, the transition from a linear shift of the tolerance zone to a vector positioning error taking into account radial and axial runout. Secondly, replacing the flat 0-0 boundary with a cylindrical coordinate surface. Thirdly, accounting for the dependence of elastic deformations on the workpiece diameter and the variable system rigidity. The authors intend to address these issues in future publications; in this paper the focus is limited to the machining of planes, where all main effects are collinear and the model has a clear analytical solution.

Another important limitation of the proposed model is that it does not explicitly take into account thermal deformations in the process system. Under real-world conditions of the long-term continuous operation of *CNC* machines, heat generation in bearing assemblies, guides, and the feed drive leads to heating of the structural elements and their thermal expansion. This causes drift of the machine’s zero point, which potentially violates **Axiom 2** (regarding the stationarity of the coordinate system of the machine). This theory assumes either using machines with active thermal compensation systems (maintaining the stationarity of the machine coordinate system with a given accuracy), or limiting the duration of continuous operation to a thermal stability period. Otherwise, thermal drift should be considered as an additional component of the positioning error in formula (4), requiring periodic refinement based on measurement data (*Bayesian* updating of digital passports). Introducing a non-stationary temperature zone into the model and developing methods for predicting thermal deformations are the subject of the authors’ further research.

However, there are some limitations of the proposed approach that should be noted. The model assumes a linear elastic response of the system and is focused on surface machining operations. The coordinate-geometric part of the model will need to be adapted for other types of machining (e.g., shaft turning or milling complex surfaces), to account for changes in the direction of force action and variable system rigidity at different points along the path. Additionally, the practical implementation of the theory requires digital passports for all elements of the process system, which represents a separate organizational and technical challenge for enterprises.

The model will be verified on a *CNC* vertical milling machine for face milling of a plane (grade 45 steel) using the following parameter ranges: $\boldsymbol{TA_{op}}$ = 0.05–0.15mm, Δ_{pos}^{max} = 0.02–0.08 mm, Δ_{tool}^{max} = 0.01–0.05 mm. A series of ten workpieces will be machined for each combination with settings based on the calculated coordinate $\boldsymbol{A_{0pos}}$. The verification criterion will be the absence of any defects in the entire series. The verification results will be presented in a separate publication after completing the full scope of experimental studies.

Conclusion

1. This paper reveals the nature of the "digital discontinuity" in pre-production engineering, which occurs due to the lack of a mathematical framework linking the digitalized design contour (*CAD*/*CAM*) with the unstable physical factors of the actual cutting process. This discontinuity can only be overcome through the deterministic theory that takes into account positioning variability and elastic deformations.

2. The existence and uniqueness of a stationary coordinate base – the «0-0 boundary» – in a *CNC* machine coordinate system has been theoretically proven for the first time. It has been established that this boundary, drawn through the highest possible position of the lower limit of the tolerance zone, is a process invariant (meaning that it remains unchanged under any permissible variations in the workpiece position within a batch). Its introduction allows transforming the non-stationary "tolerance zone – workpiece" system into a stationary one relative to the machine coordinate system, thus creating the basis for the deterministic calculation of setup parameters.

3. A fundamental law of accuracy balance, equation (2), has been formulated, establishing a criterion for the fundamental feasibility of the technological process. It is shown that if the $\boldsymbol{TA}_{op} \geq \Delta_{pos}^{\max} + \Delta_{tool}^{\max} + [\boldsymbol{\varepsilon}_{ed}]$ condition is violated, a batch of parts cannot be machined under any positioning scheme and any cutting conditions. This principle becomes a diagnostic criterion, allowing for the identification of process design errors at the pre-production engineering stage, rather than on the shop floor.

4. The **"accuracy resource" $[\boldsymbol{\varepsilon}_{ed}]$ criterion has been substantiated**, quantitatively characterizing the safety margin of the process system. It has been proved that the accuracy resource fulfills two functions: it indicates the fundamental process feasibility ($[\boldsymbol{\varepsilon}_{ed}] \geq 0$) and is used as a deterministic limiter of cutting conditions during the finishing pass ($\boldsymbol{y}_d \leq [\boldsymbol{\varepsilon}_{ed}]$). This creates an analytical basis for determining machining conditions that ensure a specified accuracy.

5. A closed system of equations (1–7) has been developed, representing a complete mathematical model for dimensional setting theory. This model establishes a direct analytical relationship between the initial process parameters and the desired absolute tool coordinate, allowing for a transition from selecting empirical parameters to their calculation at the pre-production engineering stage.

6. A methodological basis has been created for implementing unmanned technologies. It is demonstrated that the developed theory provides the theoretical foundation for eliminating the trial machining cycle, provided that reliable initial data (digital passports) are available and the model applicability conditions is met, since product quality can be guaranteed at the design stage through the deterministic calculation of tool coordinates. This paves the way for creating digital twins of machining operations and intelligent *CAM* modules that can automatically calculate setup parameters without human intervention.

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Conflicts of Interest

The authors declare no conflict of interest.