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Analytical assessment of the processing productivity of complex workpieces based on Markov models

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ABSTRACT

Introduction. In multi-product manufacturing, loading, unloading, and auxiliary movements of equipment components contribute significantly to the overall processing cycle time. It is hypothesized that transitioning from the processing of individual simple workpieces to complex workpieces – combining several parts in a single unit – can reduce unproductive time expenditures. However, existing analytical approaches do not provide a quantitative assessment of the productivity gain while accounting for the probabilistic nature of transitions between technological states. ***Purpose of the work*** is to develop analytical expressions for evaluating the productivity of a technological system when processing simple and complex workpieces on the basis of *Markov* models. The research objective is to establish a correspondence between the time parameters of conventional time-and-motion study (technical rating) and the transition intensities in a *Markov* chain, and to derive a formula for the relative gain. The technological process is represented as a directed graph, with transitions between states specified by intensities. **The research methods** include mathematical modeling based on the fundamental principles of *Markov* process theory, numerical simulation in the *Maple* environment, and the principles of group parts processing theory. **Results and discussion.** The developed mathematical models enable quantitative estimation of the fractions of time that the technological system spends in the states of loading, processing, auxiliary movements, and unloading. The analytical relationships obtained directly relate productivity to the transition intensities between states. It is shown that the key states determining processing efficiency are loading, unloading, and auxiliary movements. Numerical simulations performed in the Maple environment confirmed the validity of the derived formulas. The results obtained demonstrate the potential of the proposed approach for the analytical assessment of the processing productivity of complex workpieces. The derived expressions are valid for a wide range of surface configurations and can be used to calculate the gain achieved when combining an arbitrary number of parts.

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Introduction

In modern machine and instrument engineering, a steady trend toward increasing requirements for the productivity of metalworking equipment is observed. This is driven by the need to reduce production costs, shorten the duration of the technological cycle, and enhance the competitiveness of manufactured products. The classic piece–time formula, known from the fundamentals of technological rate setting, assumes that the manufacturing time for a single part consists of the main (machine) time, auxiliary time, workplace maintenance time, and rest time [1]. However, this model is based on the assumption of the deterministic nature of all time expenditures and does not consider the stochastic nature of real production. Real production systems operate under conditions of significant uncertainty. Unscheduled downtime, tool failures, defects, and other events of a probabilistic nature are possible [2]. Therefore, adequate analysis and productivity forecasting require mathematical methods that take these factors into account.

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The probability-theoretic approach to the analysis of production systems has a long history. The fundamentals of modeling complex systems using continuous-time *Markov* chains were laid in the works of *A.D. Wentzell* [3], as well as *E.S. Wentzel* and *L.A. Ovcharov* [4], where methods for analyzing random processes are discussed in detail. The classic textbook by *J.R. Norris* [5] provides a rigorous presentation of Markov chain theory, including methods for calculating limit probabilities. Further development in the field of simulation modeling of production systems was continued by *A.M. Law* [6]. In recent years, there has been active development of mathematical modeling methods as applied to mechanical engineering problems. In [7], a comprehensive approach to the integration of lean manufacturing tools and digital technologies (*Lean 4.0*) is proposed. The authors of [8] consider the concept of “factories of the future” in the context of *Industry 5.0*. These concepts require an adequate mathematical apparatus for evaluating the efficiency of technological solutions at the design stage.

The fundamentals of the theory of group machining of parts were laid by *S.P. Mitrofanov* [9], who introduced the concept of a complex part as a generalized model synthesizing all surfaces of the parts included in a technological group. In developing these ideas, works [10, 11] proposed the concept of using complex workpieces – the physical combination of several workpieces of a complete group into a single block for sequential machining in one setup. It was shown that this approach can significantly reduce the proportion of auxiliary time; however, analytical dependencies for quantifying the productivity gain were not obtained in these works. Probability-theoretic aspects of the functioning of technological systems were also considered in [12]. Issues of analysis and modeling of technological processes based on graph theory and *Markov* processes are discussed in [13]. A broader review of classification methods for mathematical models in technological design is presented in [14]. Issues of increasing the rigidity of the technological system during machining on metal-cutting machines are considered in the monograph [15]. The problems of controlling the machining of low-rigidity parts are addressed in [16]. An analysis of equipment changeover capability is provided in [17], and the synthesis of a technological system based on morphological analysis is performed in [18]. These studies create a theoretical basis for designing technological systems for machining complex workpieces. Works [19, 20] consider issues of predictive adaptation of process parameters when machining hybrid workpieces (from different materials) and the assessment of economic effects from product customization.

Despite the significant number of works devoted to both group technology and probabilistic modeling, the task of deriving closed-form analytical expressions for the quantitative assessment of the productivity of a technological system when machining complex workpieces remains unresolved. Most existing approaches are oriented either toward deterministic piece–time models or toward simulation modeling, which does not provide explicit formulas for calculating the productivity gain.

The purpose of this work is to develop analytical expressions for assessing the productivity of a technological system when machining simple and complex workpieces based on continuous-time *Markov* models. To achieve this aim, ***the following tasks*** were solved during the research:

– to establish correspondences between the time parameters of classical rate setting and the transition intensities in a *Markov* chain;

– to derive formulas for the productivity of a single workpiece through the limit probabilities of key states;

– to derive formulas for the productivity of a complex workpiece through the limit probabilities of key states;

– to obtain an analytical expression for the relative gain when transitioning from separate machining of single workpieces to the machining of a complex workpiece.

Materials and methods

This paper considers two types of workpieces: single and complex. A single workpiece (Fig. 1) represents an individual part and is formed using the theory of group machining, which allows for the maximum nomenclatural saturation of elementary external surfaces machined by turning [9]. This, in turn, makes it

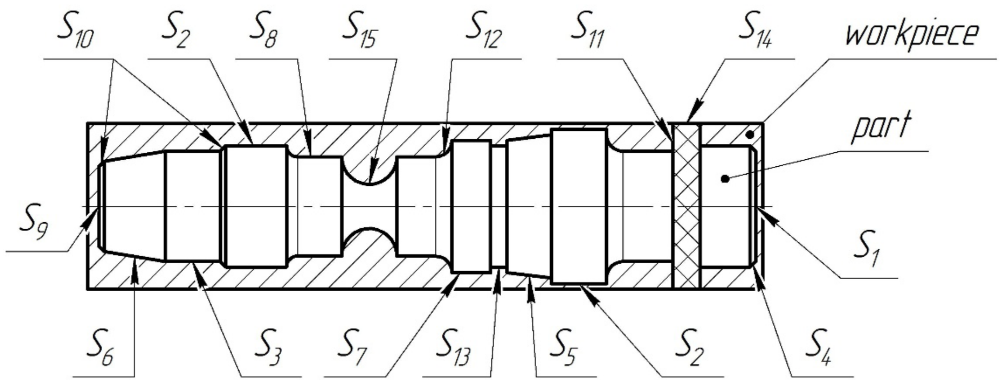


Fig. 1. Example of a single workpiece

possible to quantitatively assess and identify the greatest values of the technological system's residence time in each of the studied states [10, 11].

It should be noted that the term "complex workpiece" used in this work is not identical to the concept of "complex part" introduced by *S.P. Mitrofanov* within the framework of group technology theory [9]. In the classical understanding, a complex part is a generalized model synthesizing all surfaces of a group of parts, combining all structural elements (surfaces) of parts belonging to a technological group, and serving as the basis for developing a group technological process applicable to any part in the group. In contrast, in this work, a complex workpiece (Fig. 2) is understood as the physical combination of several separate workpieces of a complete group (***i***, ***j***, ***k***) into a single block for their simultaneous or sequential machining in one setup [10, 11].

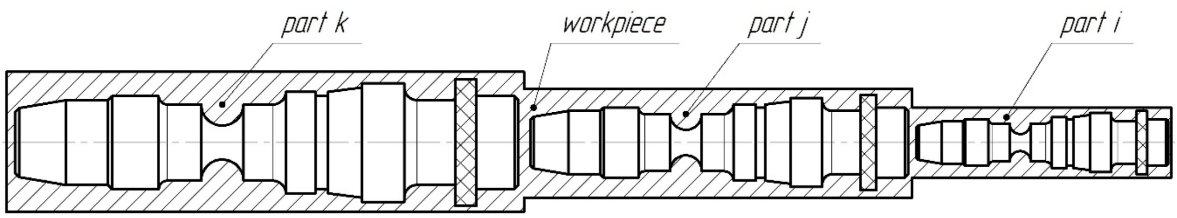


Fig. 2. Example of a complex workpiece

Besides the fact that machining from a single setup allows for achieving greater accuracy of mating parts launched into production as a complete group, a comparison of two technological approaches – machining individual single workpieces and machining complex workpieces – shows that in the second case, the time for auxiliary operations, as well as loading and unloading operations, is significantly reduced, which in theory should lead to increased productivity.

The developed analytical productivity model is based on the results of preliminary studies of the concept of using complex workpieces in production, aimed at increasing their flexibility and productivity, in which the structure and nomenclatural saturation of the surfaces of the machined workpieces were determined [10, 11].

In Fig. 1, the following designations of states corresponding to various technological turning transitions are adopted: S_1 – facing; S_2 – turning of an open cylindrical surface; S_3 – turning of a semi-open cylindrical surface; S_4 – turning of an open right-hand chamfer; S_5 – turning of a semi-open conical surface; S_6 – turning of an open conical surface; S_7 – turning of a semi-open cylindrical surface bounded by a semi-open cone; S_8 – turning of a semi-open cylindrical surface bounded by a fillet; S_9 – parting off the part; S_{10} – turning of a left-hand chamfer; S_{11} – facing of a semi-open face; S_{12} – turning of a fillet; S_{13} – turning of a groove without reference to type and size; S_{14} – knurling; S_{15} – turning of a shaped surface.

Based on the analysis of the geometry and technological transitions for each type of workpiece, directed graphs of the technological process were constructed [10, 11].

This paper considers a *Markov* process describing the functioning of an ideal turning technological process. The model assumes the absence of failures, breakdowns, and defects, which allows for estimating the upper bound of the system's productivity. The state space is divided into two classes: main technological transitions (material removal) and auxiliary transitions (loading, unloading, tool approach/retraction, and tool change), which makes it possible to determine the proportion of machine time and obtain limiting efficiency estimates.

The constructed graphs belong to the class of pure birth *Markov* processes, which fully corresponds to the irreversible nature of the technological route, where each transition is performed once and return to the previous state is not provided for. In this class of processes, transitions are possible only from a state with a smaller number to a state with a larger number, while reverse transitions (death) and absorbing states (failures) are absent. According to the classic works of *E.S. Wentzel* and *L.A. Ovcharov*, such a process is a special case of birth-and-death processes with zero "death" intensities [4].

For the assessment of productivity, auxiliary transitions are of key importance, as they determine the proportion of non-productive time expenditures in the overall cycle. Considering the cumbersomeness of the full graphs, which include all possible states and transitions, their simplified structures are presented in this work (Fig. 3). The vertices of the graph correspond to individual states corresponding to various technological turning transitions, and the directed edges represent possible transitions between them with corresponding intensities λ_n^m.

In general form, the transition intensity matrix $\mathbf{\Lambda}$ has the dimension $[\boldsymbol{n} \times \boldsymbol{n}]$, where $\boldsymbol{n}$ is the total number of system states. The elements λ_n^m of the matrix represent the intensities of transition from state $\boldsymbol{n}$ to state $\boldsymbol{m}$. The diagonal elements are determined from the condition of probability sum conservation:

$$\lambda_n^n = -\sum_{n \neq m} \lambda_n^m .$$

For a single workpiece (Fig. 1), a graph containing 18 vertices was obtained. Of these, 15 vertices correspond to the surface machining states $\boldsymbol{P_1}, \boldsymbol{P_2}, \ldots \boldsymbol{P_n}$ ($\boldsymbol{n}$ = 15) and three vertices correspond to the key states: loading $\boldsymbol{P_{16}}$, unloading $\boldsymbol{P_{17}}$, and auxiliary movements $\boldsymbol{P_{18}}$. Correspondingly, the transition intensity matrix $\mathbf{\Lambda}_{single}$ has the dimension [18 × 18].

For a complex workpiece (Fig. 2), which combines $\boldsymbol{m}$ single workpieces, the transition graph is constructed as the *Cartesian* product of the graphs of the individual workpieces with the addition of inter-workpiece transitions. If each single workpiece is described by 18 states, then with independent consideration of three workpieces without taking into account common states, the number of states would be

$$\boldsymbol{N_{complex}} = (\boldsymbol{N_{single}})^{\boldsymbol{m}} = 18^3 = 5{,}832.$$

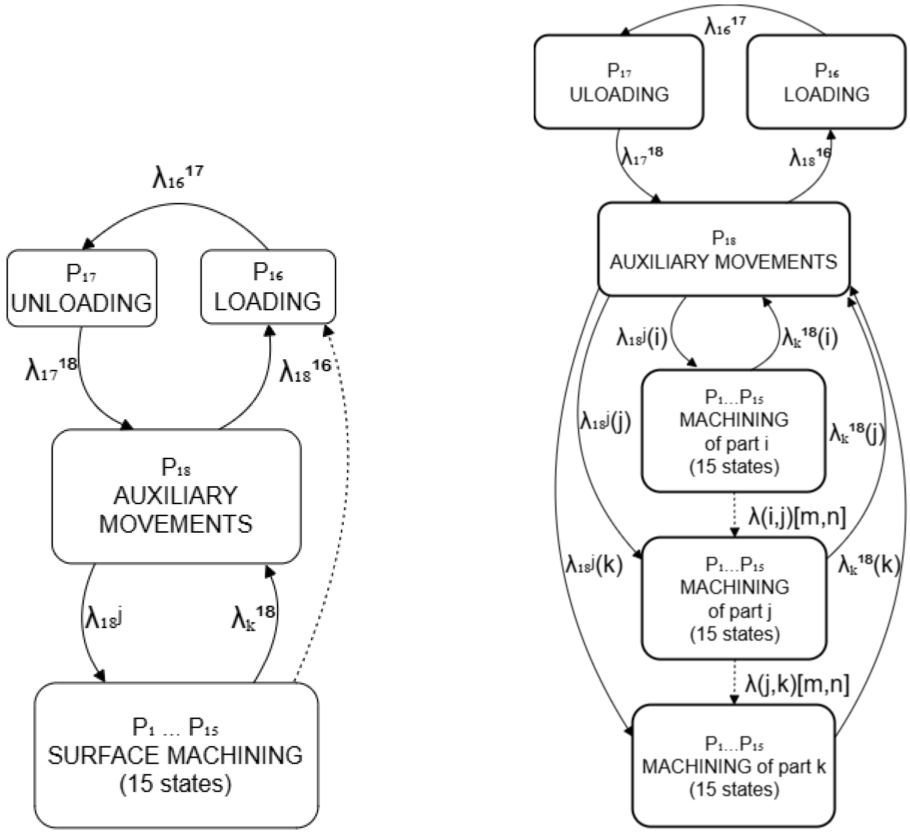
However, in a real technological system, the states of loading, unloading, and auxiliary operations are common to all parts of the complex workpiece. Therefore, the repeating key states are excluded from the *Cartesian* product. The total number of states for a complex workpiece is determined by the formula:

$$\boldsymbol{N_{complex}} = \boldsymbol{m} \times \boldsymbol{N_{proc}} + \boldsymbol{N_{auxil}} = 3 \times 15 + 3 = 48.$$

Correspondingly, the transition intensity matrix $\mathbf{\Lambda}_{complex}$ has the dimension [48 × 48]. For a clear representation of the structure of the technological process, simplified transition graphs for a single and a complex workpiece are shown in Fig. 3.

Let us consider a continuous-time *Markov* chain describing the technological process of manufacturing a part. The system can be in n different states. In the stationary mode, for each state $\boldsymbol{n}$, the balance equation is fulfilled [3]:

$$P_n \sum_{m \neq n} \lambda_n^m = \sum_{m \neq n} P_m \lambda_m^n, n = 1, 2, .., k, \tag{1}$$



a *b*

Fig. 3. Simplified structures of transition graphs:
a – for a single workpiece; *b* – for a complex workpiece

where λ_m^n is the transition intensity from state $\boldsymbol{n}$ to state $\boldsymbol{m}$; $\boldsymbol{P_n}$, $\boldsymbol{P_m}$ are the flow probabilities transferring the system from state $\boldsymbol{S_n}$ to state $\boldsymbol{S_m}$.

System (1) is supplemented by the normalization condition:

$$\sum_{n=1} P_n = 1\ (n = \overline{1,k}). \tag{2}$$

In the model of the technological process, it is possible to distinguish key states that directly affect actual productivity: $\boldsymbol{P_{16}}$ – loading state; $\boldsymbol{P_{17}}$ – unloading state; $\boldsymbol{P_{18}}$ – state of auxiliary movements (idle travels, tool approaches); $\boldsymbol{P_1}$ … $\boldsymbol{P_{15}}$ – surface machining states. The transition intensities between these states are denoted as λ_n^m.

For a correct description of the process, the following relations must be fulfilled: from the loading state, only a transition to the unloading state is possible: $\boldsymbol{\lambda_n^m} > 0$, the rest $\lambda_{16}^m > 0$; from the unloading state, only a transition to the auxiliary movements state is possible: $\lambda_{17}^{18} > 0$; from the auxiliary movements state, transitions to machining states and to the loading state are possible.

Let us write the balance equations (1) for states $\boldsymbol{P_{16}}$, $\boldsymbol{P_{17}}$ and $\boldsymbol{P_{18}}$. For the loading state $\boldsymbol{P_{16}}$, the incoming flows are only from $\boldsymbol{P_{18}}$ with intensity λ_{18}^{16}, and the outgoing flows are only to $\boldsymbol{P_{17}}$ with intensity λ_{16}^{17}, giving the balance equation:

$$P_{16}\lambda_{16}^{17} = P_{18}\lambda_{18}^{16}. \tag{3}$$

For the unloading state $\boldsymbol{P_{17}}$, the incoming flows are only from $\boldsymbol{P_{16}}$ with intensity λ_{16}^{17}, and the outgoing flows are only to $\boldsymbol{P_{18}}$ with intensity λ_{17}^{18}, so the balance equation will be:

$$P_{17}\lambda_{17}^{18} = P_{16}\lambda_{16}^{17}. \tag{4}$$

For the auxiliary movements state $\boldsymbol{P_{18}}$, the incoming flows arrive from all machining states $\boldsymbol{P_n}$ ($\boldsymbol{n}$ = 15) with intensities λ_{18}^{m} and from the unloading state $\boldsymbol{P_{17}}$ with intensity λ_{17}^{18}. The outgoing flows are directed to all machining states $\boldsymbol{P_m}$ ($\boldsymbol{m}$ = 15) with intensities λ_{18}^{m} and to the loading state P_{16} with intensity λ_{18}^{16}. The balance equation in this case is:

$$P_{18}\left(\sum_{m=1}^{15}\lambda_{18}^{m} + \lambda_{18}^{16}\right) = \sum_{n=1}^{15} P_n\lambda_n^{18} + P_{17}\lambda_{17}^{18}. \tag{5}$$

From equations (3) and (4), the relation between probabilities is obtained:

$$P_{16} = \frac{\lambda_{18}^{16}}{\lambda_{16}^{17}}P_{18}, \qquad P_{17} = \frac{\lambda_{18}^{16}}{\lambda_{16}^{17}}P_{18}. \tag{6}$$

To determine productivity, it is necessary to establish the average cycle time – a key quantity characterizing the duration of a complete processing cycle. In classical technical rate setting, the manufacturing time of a single part (piece time $\boldsymbol{T_p}$) includes fixed components, including loading time $\boldsymbol{T_{ld}}$, unloading time $\boldsymbol{T_{ul}}$, and other auxiliary operations time $\boldsymbol{T_{aux}}$. However, in a real stochastic process, the duration of each of these operations is a random variable.

The average cycle time $\boldsymbol{T_c}$ is the sum of all stages of machining one part (or a set of parts):

$$T_c = \tau_{ld} + \sum_{k=1}^{n}\tau_{mt}^{(n)} + \tau_{aux} + \tau_{ul}$$

where $\boldsymbol{\tau_{ld}}$ is the average loading time; $\boldsymbol{\tau_{mt}}$ is the average machining time of the n-th surface; $\boldsymbol{\tau_{aux}}$ is the average auxiliary operations time; $\boldsymbol{\tau_{ul}}$ is the average unloading time.

Since the technological system sequentially passes through the key states $\boldsymbol{P_{16}}$ (loading), $\boldsymbol{P_{17}}$ (unloading), and $\boldsymbol{P_{18}}$ (auxiliary movements), returning to the initial state after completing a full cycle, the average cycle time can be expressed through the probability of the system being in each of these states and the intensities of exit from them. This approach takes into account the stochastic nature of the process and allows linking time characteristics with the limit probabilities of states [4, 7].

Let us find the average cycle time $\boldsymbol{T_c}$ through the loading state $\boldsymbol{P_{16}}$. In the stationary mode, the average time between two consecutive loading acts is inversely proportional to the frequency of loading completion. Since, on average, $P_{16}\times\lambda_{16}^{17}$ "loading completed" events occur per unit time, the average cycle time will be:

$$T_c = \frac{1}{P_{16}\cdot\lambda_{16}^{17}} \tag{7}$$

Considering that the average loading time $\tau_{ld} = 1/\lambda_{16}^{17}$ corresponds to the classical time $\boldsymbol{T_{ld}}$, we obtain: $\boldsymbol{T_c} = \tau_{ld}/P_{16}$.

Through the unloading state $\boldsymbol{P_{17}}$. Similarly, unloading is a mandatory final stage of the cycle. The frequency of unloading completion is $P_{17}\times\lambda_{17}^{18}$. The average cycle time through the unloading state is:

$$T_c = \frac{1}{P_{17}\cdot\lambda_{17}^{18}} = \frac{\tau_{ul}}{P_{17}}, \tag{8}$$

where $\tau_{ul} = 1/\lambda_{17}^{18}$ is the average unloading time, corresponding to the classical time $\boldsymbol{T_{ul}}$.

Let us find the average cycle time $\boldsymbol{T_c}$ through the auxiliary movements state $\boldsymbol{P_{18}}$. The state $\boldsymbol{P_{18}}$ is a system-forming element through which transitions between machining and loading/unloading are realized. The frequency of exit from this state is $P_{18} \sum_{m \in M} \lambda_{18}^{m}$, where $\boldsymbol{M}$ is the set of states into which a transition from $\boldsymbol{P_{18}}$ is possible (machining states and the loading state). The average cycle time through this state is:

$$T_c = \frac{1}{P_{18} \sum_{m \in M} \lambda_{18}^{m}} = \frac{\tau_{aux}}{P_{18}}, \tag{9}$$

where $\tau_{aux} = 1 \Big/ \sum_{m \in M} \lambda_{18}^{m}$ is the average time spent in the auxiliary movements state per visit, which corresponds to the classical auxiliary operations time $\boldsymbol{T_{aux}}$.

Thus, the average cycle time can be expressed through any of the key states, and all three expressions are equivalent:

$$T_c = \frac{1}{P_{16}\lambda_{16}^{17}} = \frac{1}{P_{17}\lambda_{17}^{18}} = \frac{1}{P_{18} \sum_{m \in M} \lambda_{18}^{m}} = \frac{\tau_{ld}}{P_{16}} = \frac{\tau_{ul}}{P_{17}} = \frac{\tau_{aux}}{P_{18}}. \tag{10}$$

From equations (10), it follows that the mean cycle time $\boldsymbol{T_c}$ is fully determined by the state probabilities of the key stages and the corresponding transition intensities. Given that productivity is defined as the inverse of the cycle time $\boldsymbol{Q} = 1 / T_c$, the general productivity expression is obtained directly from the derived relationships. By recasting the time-based parameters in terms of the number of workpieces machined per unit time, the following result is obtained.

Productivity for a single workpiece (Fig. 1):

$$Q_s = \frac{1}{T_c} = P_{16}^{s}\lambda_{16}^{17} = \frac{P_{16}^{s}}{\tau_{ld}}. \tag{11}$$

The obtained expression shows that the productivity of a single workpiece is directly proportional to the probability of the loading state and inversely proportional to the average loading time. This formula is a generalization of the classical relation $\boldsymbol{Q} = 1 / T_p$, since in the absence of random variations and at $P_{16}^{s} = 1$ it tends to the deterministic limit.

For a clear representation of the correspondence between classical time parameters and the elements of the *Markov* model (Fig. 4), a correspondence diagram is provided: the left part of the diagram illustrates the classical deterministic model, while the right part shows the Markov model, where each classical time is matched with an average value.

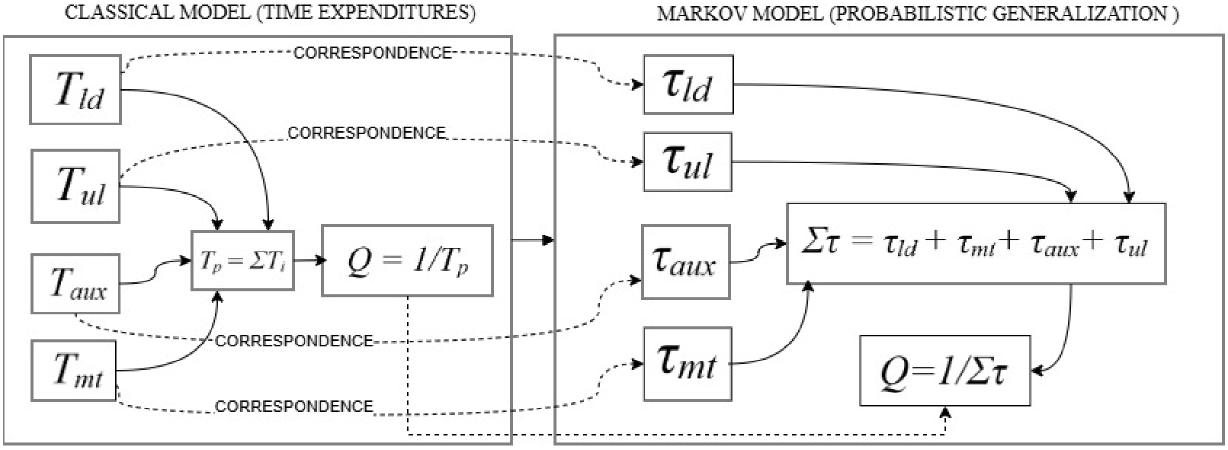


Fig. 4. Correspondence chart between the classical and *Markov* productivity models

For a complex workpiece containing ***m*** parts, ***m*** parts are machined in one cycle while maintaining the same cycle time (loading and unloading are performed once). The productivity per part is:

$$Q_{complex} = \frac{m}{T_c} = mP_{16}^{complex}\lambda_{16}^{17} = m\frac{P_{16}^{complex}}{\tau_{ld}}.. \tag{12}$$

Comparison of expressions (11) and (12) shows that the gain in productivity when transitioning to a complex workpiece is determined by the ratio of the loading probabilities in the two models and the number of combined parts. Considering that the proportionality of the loading and auxiliary state probabilities follows from the balance equations (6), the gain can be expressed through the probability ratio $P_{18}^{complex}$ and P_{18}^{s}:

$$\frac{Q_{complex}}{Q_s} = m \cdot \frac{P_{18}^{complex}}{P_{18}^{s}} \tag{13}$$

At the same time, to find $\boldsymbol{P_{18}}^{complex}$ and $\boldsymbol{P_{18}}^{s}$, we use the balance equation (5) for the auxiliary movements state, obtaining:

$$P_{18}^{s} = \frac{\sum_{n=1}^{15} P_n\lambda_n^{18} + P_{17}\lambda_{17}^{18}}{\sum_{m=1}^{15} \lambda_{18}^{m} + \lambda_{18}^{16}}, \qquad P_{18}^{complex} = \frac{\sum_{x\in\{i,j,k\}}\sum_{n=1}^{15} P_n(x)\lambda_n^{18}(x) + P_{17}\lambda_{17}^{18}}{\sum_{x\in\{i,j,k\}}\sum_{m=1}^{15} \lambda_{18}^{m}(x) + \lambda_{18}^{16}}. \tag{14}$$

The expressions (14) represent the total incoming flow into state $\boldsymbol{P_{18}}$, which is composed of the flow from all machining states $\sum_{n=1}^{15} P_n\lambda_n^{18}$ and the flow from the unloading state $P_{17} \times \lambda_{17}^{18}$. The denominator represents the total outgoing flow from state $\boldsymbol{P_{18}}$, which is composed of the flow to all machining states: $\sum_{m=1}^{15} \lambda_{18}^{m}$ and the flow to the loading state: λ_{18}^{16}.

Let us introduce the following notation for the total flows:

$S = \sum_{n=1}^{15} P_n\lambda_n^{18} = \sum_{n=1}^{15} \frac{P_n}{\tau_{mt}^{(n)}}$ – total flow from machining states to state $\boldsymbol{P_{18}}$; $R = \sum_{m=1}^{15} \lambda_{18}^{m} = \sum_{m=1}^{15} \frac{1}{\tau_{mt}^{(m)}}$ – total flow from state $\boldsymbol{P_{18}}$ to machining states; $Z = P_{17}\lambda_{17}^{18} = \frac{P_{17}}{\tau_{ul}}$ – flow from unloading to state $\boldsymbol{P_{18}}$; $\lambda_{18}^{16} = \frac{1}{\tau_{aux}}$ – intensity of transition from state $\boldsymbol{P_{18}}$ to the loading state.

Then expressions (14) for the probability of the auxiliary state take a compact form, considering that for a complex workpiece combining ***m*** identical parts, the total flows from and to the machining states increase proportionally to ***m***, while the flow from unloading remains unchanged (unloading is common). We obtain:

$$P_{18}^{s} = \frac{S+Z}{R+\lambda_{18}^{16}}, \qquad P_{18}^{complex} = \frac{mS+Z}{mR+\lambda_{18}^{16}}. \tag{15}$$

Substituting (16) and (17) into relation (13), we obtain the final analytical expression for the relative gain:

$$\frac{Q_{complex}}{Q_s} = m\frac{mS+Z}{mR+\lambda_{18}^{16}}\frac{R+\lambda_{18}^{16}}{S+Z} = m\frac{m\sum_{n=1}^{15}\frac{P_n}{\tau_{mt}^{(n)}} + \frac{P_{17}}{\tau_{ul}}}{m\sum_{m=1}^{15}\frac{1}{\tau_{mt}^{(m)}} + \frac{1}{\tau_{aux}}}\frac{\sum_{m=1}^{15}\frac{1}{\tau_{mt}^{(m)}} + \frac{1}{\tau_{aux}}}{\sum_{n=1}^{15}\frac{P_n}{\tau_{mt}^{(n)}} + \frac{P_{17}}{\tau_{ul}}}. \tag{15}$$

In the limiting cases, when machining dominates (***S*** >> ***Z*** and ***R*** >> λ_{18}^{16}) $\dfrac{P_{18}^{complex}}{P_{18}^{s}} \approx \dfrac{mS}{mR} \cdot \dfrac{R}{S} = 1$, hence $\dfrac{Q_{complex}}{Q_s} \approx m$ the gain by ***m*** a factor of is achieved due to one loading operation serving ***m*** parts, while the machining time is proportional to the number of parts. In the case when loading/unloading dominates (***Z*** >> ***S*** and λ_{18}^{16} >> ***R***) $\dfrac{P_{18}^{complex}}{P_{18}^{s}} \approx \dfrac{Z}{\lambda_{18}^{16}} \dfrac{\lambda_{18}^{16}}{Z} = 1$, similarly $\dfrac{Q_{complex}}{Q_s} \approx m$, the gain tends to ***m***, i.e., to the number of combined parts.

Results and discussion

To verify the obtained analytical relations and quantify the efficiency of complex workpieces, numerical modeling was performed in *Maple*. The initial data were vectors of limit state probabilities obtained from solving systems of linear algebraic equations for single and complex workpieces (Table 1). In accordance with the problem conditions, all non-zero transition intensities λ_n^m were taken equal to 1, which corresponds to the equal duration of all operations in relative time units [10, 11].

Table 1

Values of the limiting probabilities of the states of single and complex workpieces

No.	Probability P_i	Single workpiece	Complex workpiece		
			Workpiece 1 (***i***)	Workpiece 2 (***j***)	Workpiece 3 (***k***)
1	P_1	0.0015026	0.0010172	0.0010172	0.0010172
2	P_2	0.0582622	0.0028241	0.0030031	0.0444621
3	P_3	0.1867397	0.0035301	0.0037539	0.1333865
4	P_4	0.0086120	0.0022523	0.0011321	0.0049053
5	P_5	0.0995299	0.0206797	0.0172451	0.0654489
6	P_6	0.0715674	0.0048921	0.0052168	0.0510131
7	P_7	0.1867397	0.0282414	0.0300313	0.1333865
8	P_8	0.15241673	0.0282414	0.0300313	0.1333865
9	P_9	0.0754756	0.0138810	0.0123473	0.0544353
10	P_{10}	0.0036893	0.0019013	0.0001886	0.0008175
11	P_{11}	0.0415682	0.0036925	0.0038947	0.0294997
12	P_{12}	0.0343229	0.0045620	0.0048465	0.0232694
13	P_{13}	0.01352405	0.0045777	0.0068666	0.0160222
14	P_{14}	0.0135240	0.0045777	0.0068666	0.0160222
15	P_{15}	0.0119529	0.0007682	0.0024885	0.0108900
16	$P_{16}(P_{00})$	**0.0135240**	**0.0091555**		
17	$P_{17}(P_{00})$	**0.0135240**	**0.0091555**		
18	$P_{18}(P_0)$	**0.01352405**	**0.0091555**		

As a result of solving the systems of equations, the values of the limit probabilities of the key states were obtained, based on which the time characteristics and productivity for both types of workpieces were calculated. The summary results are presented in Table 2.

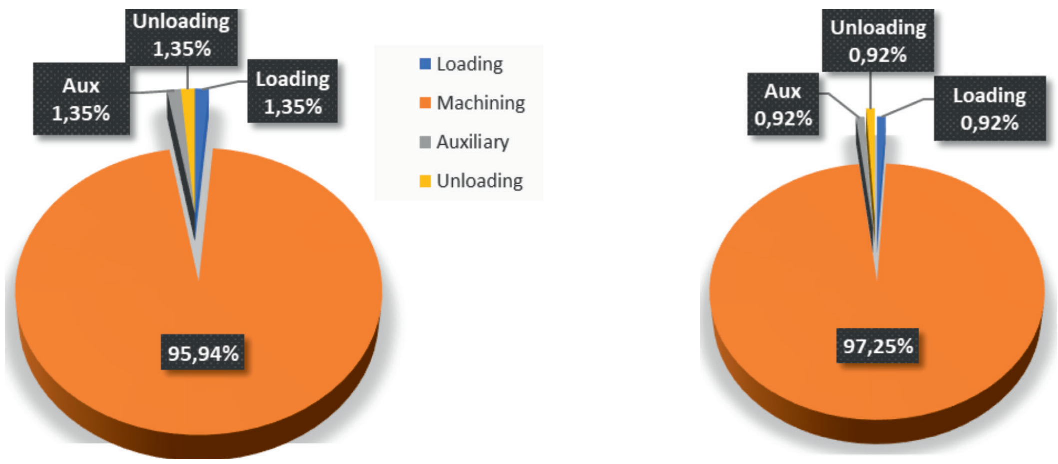
Table 2

Comparison of the productivity of single and complex workpieces

Parameter	Single workpiece	Complex workpiece
Loading probability, P_{16}	0.013524	0.009156
Unloading probability, P_{17}	0.013524	0.009156
Auxiliary operations probability, P_{18}	0.013524	0.009156
Average cycle time per part, T_{cyc} (min)	73.94	36.41
Productivity, Q (parts/h)	0.811	1.648

Analysis of the presented data allows the following conclusions to be drawn. When transitioning from separate machining of single workpieces to a complex workpiece, the probabilities of loading, unloading, and auxiliary operations decrease by a factor of approximately 1.48, reflecting a reduction in the proportion of non-productive time in the overall cycle. The productivity of the complex workpiece more than doubles compared to the separate machining productivity: 1.648 parts/h versus 0.811 parts/h. The productivity ratio is 2.031, corresponding to a gain of 103.1%.

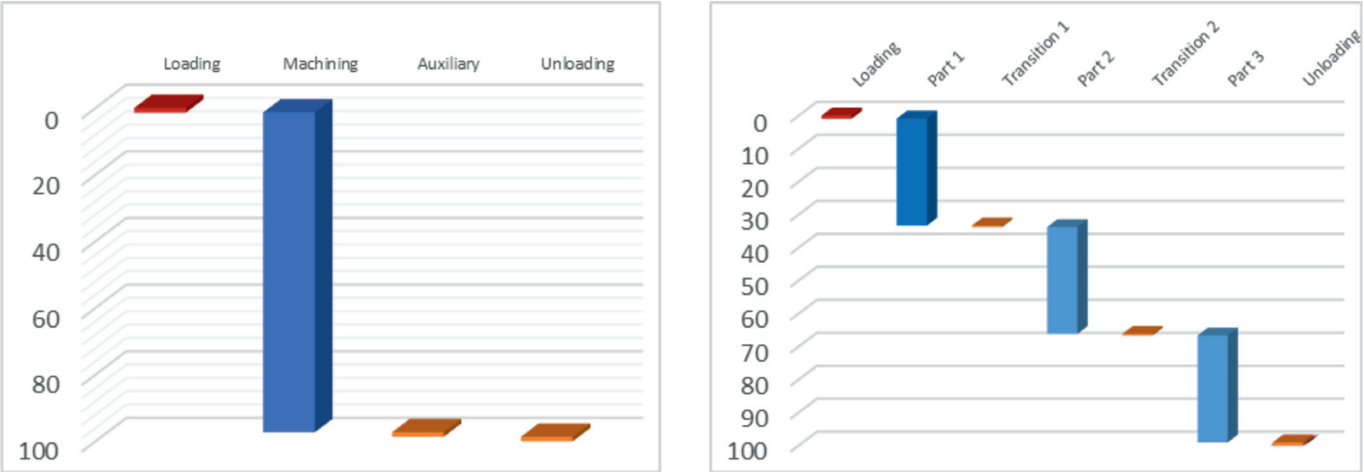
For a clear representation of the cycle time distribution, diagrams and *Gantt* charts were constructed based on the calculated limit probabilities. Fig. 5,*a* presents a diagram for a single workpiece, clearly showing that the main share of time (more than 95%) is spent on machining, while loading, unloading, and auxiliary operations occupy an insignificant part of the cycle – 1.35% each. In the diagram for a complex workpiece of three parts (Fig. 5, *b*), it can be seen that the share of time for machining increases to 96.2%, while the shares of loading, unloading, and auxiliary operations decrease to 0.97% each, indicating an increase in equipment utilization efficiency.



a *b*

Fig. 5. Distribution diagrams of the manufacturing cycle time:
a – for a single workpiece; *b* – for a complex workpiece comprising three single workpieces

The *Gantt* charts presented in Fig. 6 show the sequence of operations over time. For a single workpiece (Fig. 6, *a*), four sequential stages are clearly visible: loading, machining, auxiliary operations, and unloading. For the complex workpiece (Fig. 6, *b*), the *Gantt* chart demonstrates that loading and unloading are performed once for the entire set of parts, and machining is distributed among the three parts, separated by brief transitions.



a *b*

Fig. 6. Gantt charts:
a – for a single workpiece; *b* – for a complex workpiece

The percentage ratio of time shares, reflected in the diagrams, allows the following practical recommendations to be formulated:

1. When designing complex workpieces, one should strive for an arrangement of parts that minimizes idle tool movements between the surfaces of different parts.

2. When choosing cutting modes, it is necessary to consider their influence on the intensities $\boldsymbol{S}$ and $\boldsymbol{R}$. Increasing the cutting speed can reduce machining time but may increase the frequency of tool changes, which will increase $\boldsymbol{S}$.

3. When organizing production, one should strive for the maximum loading intensity λ_{18}^{16}, as productivity is directly proportional to this parameter.

4. To assess efficiency, it is sufficient to monitor just three probabilities: $\boldsymbol{P_{16}}$, $\boldsymbol{P_{17}}$ and $\boldsymbol{P_{18}}$, as well as the transition intensities between them. This significantly simplifies the production monitoring system.

Conclusion

1. A correspondence between the time parameters of classical technical rating (piece time, loading, unloading, and auxiliary operation time) and the transition intensities in a *Markov* chain has been established. It is shown that the average cycle time is uniquely expressed through the probabilities of the key states and the intensities of exit from them, and the transition intensities are interpreted as quantities reciprocal to the average operation execution times. This relationship substantiates the applicability of the *Markov* approach for analyzing the productivity of technological systems.

2. An analytical expression for the productivity of a single workpiece has been derived, according to which it is expressed through the probability of the loading state and the intensity of the transition from loading to unloading. However, considering the balance equations, productivity can also be expressed through the unloading or auxiliary movement states, which indicates the equivalence of these three states for efficiency assessment. Numerical modeling showed that with equal intensities of all transitions ($\boldsymbol{\lambda} = 1$), the probabilities of loading, unloading, and auxiliary operations are identical and equal to $P_{16}^{s} = P_{17}^{s} = P_{18}^{s} = 0.0135$, and the productivity of a single workpiece is $\boldsymbol{Q_s} = 0.811$ parts/h.

3. An analytical expression for the productivity of a complex workpiece combining several parts has been obtained. It is shown that the productivity per part increases proportionally to the number of parts in the set, despite the decrease in the probabilities of the key states (for a set of three parts $P_{16}^{complex} = P_{17}^{complex} = P_{18}^{complex} = = 0.00916$). Numerical modeling for $\boldsymbol{m} = 3$ yielded a productivity of

$\boldsymbol{Q}_{complex}$ = 1.648 parts/h, which is 2.03 times higher than for a single workpiece. The increase in productivity is achieved because loading, unloading, and auxiliary movement operations are performed once for the entire set of parts.

4. An analytical expression for the relative gain when transitioning from separate machining of single workpieces to the machining of a complex workpiece has been derived. It is shown that the gain is determined by the ratio of the probabilities of the key states (loading, unloading, or auxiliary movements) for the two types of workpieces. Numerical modeling for a set of three parts showed a gain of $\boldsymbol{K}$ = 2.03 (a 103% increase in productivity). The proposed relationship allows the expected effect of implementing complex workpieces to be quantitatively assessed at the stage of technological preparation of production.

5. As a direction for further research, a transition from the analytical model to a simulation model taking into account the stochastic nature of real production is proposed. The analytical calculations presented in this paper were performed for ideal conditions and characterize the maximum achievable level of productivity (the upper bound). However, real production systems are subject to random disturbances – equipment failures, tool breakages, and defects. In this regard, it is planned to expand the developed state graph by introducing vertices reflecting these phenomena. Based on the expanded model, a simulation model will be constructed that will provide realistic productivity estimates (the lower bound) and enable a comparative analysis with the analytical results, thereby making it possible to quantitatively assess productivity losses due to random factors.

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Conflicts of Interest

The authors declare no conflict of interest.