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Dependence of tool wear on evolutionary changes in parameters of the dynamic cutting system

Valery Gvindjiliya [a, *], *Vilor Zakovorotny* [b]

Don State Technical University, 1 Gagarin square, Rostov-on-Don, 344000, Russian Federation

[a] https://orcid.org/0000-0003-1066-4604, vvgvindjiliya@donstu.ru; [b] https://orcid.org/0000-0003-2187-9897, vzakovorotny@dstu.edu.ru

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ABSTRACT

Introduction. A fundamental property of a dynamic cutting system (*DCS*) is its evolutionary change, caused by irreversible energy transformations in the machining zone. Within this process, the progression of tool wear acts as one of the factors influencing the evolution, while simultaneously being dependent on the state of the *DCS*. Consequently, wear depends on the parameters of the *DCS*, and the progression of wear alters the parameters of the dynamic coupling formed by cutting. In this interconnected, and therefore self-organizing, system, the intensity of wear becomes dependent on the initial parameters of the DCS, and the wear trajectory becomes dependent on the initial parameters of the system. However, to date, wear has not been considered as part of an interconnected set of subsystems. ***Subject.*** This study examines tool wear as an integral part of an evolutionarily changing cutting system, characterized by the interplay between its coordinates and wear. In particular, it investigates the patterns of change in wear trajectories as a function of the initial parameters of the cutting system. The aim of this study is to investigate the effect of irreversible energy transformations on tool wear in order to align it with the parameters of the DCS. **Methods.** This paper presents the results of mathematical modelling of the evolutionary changes in the properties of a *DCS*, which are determined by the phase trajectory of the power of irreversible energy conversions based on the work done. It examines a *DCS* model in which wear is described using *Volterra's* equation, discusses the challenges of parameterizing the equations, presents the results of numerical modelling, and provides a practical example. **Results and Discussion.** The results of modelling the relationship between wear and *DCS* parameters, and control parameters (namely, machining conditions), are presented and discussed. An example is given of how the wear behavior changes during the turning of *0.1 C–1.0 Mn-2.0 Ni-0.3 Cu-0.7 V* steel. It is demonstrated that the evolutionary properties of wear can be highly sensitive to small variations in the initial *DCS* parameters, disturbances, and control parameters. The problem of selecting the optimal cutting speed trajectory based on the work done by the forces is also considered, as well as the use of the developed mathematical tools to create a digital twin of the machining process.

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Introduction

Numerous studies have been devoted to the dynamics of the cutting process (*DCP*), with a particular focus on the stability analysis of tool motion trajectories [1–7]. Various mechanisms of stability loss have been considered, including regenerative self-excitation [1, 2], for which a simplified stability loss criterion was proposed in [3]. Self-excitation has also been associated with delays in force variations arising from changes in deformations [4–6]. It has been demonstrated that when stability is lost due to nonlinear interactions incorporated into the model, attracting sets of deformations emerge, such as limit cycles in the case of regenerative self-excitation [7, 8] and chaotic attractors [9, 10]. The study of self-excited chatter [11–13] and the conditions for the formation of chaotic attractors [14, 15] has also been extended to stability

*** Corresponding author**
Gvindjiliya Valery E., Ph.D. (Engineering), Associate Professor
Don State Technical University,
1 Gagarin square,
344000, Rostov-on-Don, Russian Federation
Tel.: +7 918 583-23-33, **e-mail:** vvgvindjiliya@donstu.ru

loss of equilibrium induced by force delays relative to deformations. Problems of self-excited chatter originating from nonlinear friction forces on the tool's rake and flank faces have been addressed [16–19]. The stability of equilibrium and the analysis of attracting sets of deformations have been examined not only in turning but also in milling [20–23] and drilling [24, 25]. A comprehensive review of research on *DCP* is presented in the monograph [26]. It should be noted that in virtually all of these studies, an autonomous dynamic cutting system (*DCS*) is considered in variations about equilibrium. In the last decade, however, studies have been published that investigate non-autonomous *DCS*s subjected to periodic excitations. These works examine the influence of periodic perturbations on the geometric topology of the machined surface [27–29]. It has been shown that the *DCP* affects not only the resulting surface topography but also the tool wear rate [30]. Moreover, the progression of wear is accompanied by changes in vibration characteristics. Consequently, the *DCP* has also been explored in the context of developing vibroacoustic diagnostic systems for wear monitoring [31–36]. Changes in cutting forces [36], vibrations [32–34], and acoustic emission [35] have been analyzed. Wear is known to result from a multitude of physical interactions accompanying the machining process [37–40]. As knowledge of wear mechanisms has evolved, a hypothesis has emerged that the wear rate is associated with irreversible energy transformations (*IET*) in the cutting zone [41, 42]. Numerous experiments have been conducted, confirming the validity of this concept [43, 44]. It was found that as wear progresses, changes in vibrational characteristics occur [42], including bifurcations of attracting sets of deformations [43]. Finally, a hypothesis was formulated regarding the existence of technological parameters that are optimal from the standpoint of tool wear resistance, determined by the trajectories of the machine tool actuating elements (*MTAE*). These parameters correspond to an optimal power of irreversible energy transformations in the cutting zone, estimable, for instance, by temperature [47–49]. Consequently, it has been proposed to select and maintain cutting conditions at which the temperature in the cutting zone corresponds to the optimal value. The existence of such conditions, particularly cutting speeds, at which the wear rate is minimized is attributed to the fact that as the power of *IET* varies, the prevailing physical wear mechanisms also change [49–54], encompassing fatigue, frictional, adhesive, diffusion, and tribochemical interactions. This concept underpins, for example, the determination of rational technological regimes at *Sandvik Coromant* [55].

The foregoing analysis shows, first, that the current tool wear results from intra-system interactions between the tool and the workpiece, mediated by the cutting process. These interactions are of different physical natures; however, their primary source lies in the work and power of irreversible energy transformations (*IET*) of the mechanical system. Consequently, the properties of the dynamic cutting system (*DCS*) are of fundamental importance in these interactions. Second, the wear rate depends on two competing processes: the adaptation of tool interactions to the cutting process, and the degradation of properties manifested in wear development. All interactions in the *DCS* are interconnected and may act synergistically toward the functional objective of minimizing the wear rate. Third, the generator of the wear rate is not the instantaneous value of the power of *IET*, but rather the entire trajectory of the power of *IET* of the mechanical system with respect to the work performed. Finally, wear development is but one manifestation of the evolutionary change in the properties of the cutting process, driven by *IET* in the cutting zone. Wear and the parameters of the dynamic interaction formed by the cutting process are functionally related and give rise to phenomena of self-organization and internal rearrangement of *DCS* properties. However, the influence of the dynamics of intra-system interrelations, accounting for the wear process and based on irreversible energy transformations in the *DCS*, has remained virtually unexplored.

In this regard, ***the present paper aims*** to reveal the interdependence between wear and the parameters of the *DCS*, whose evolution depends on the "power–work" trajectory of irreversible energy transformations in the cutting zone. This also opens up a previously unexplored avenue for improving tool wear resistance through the synergetic alignment of the evolutionary properties of the wear rate and *DCS* parameters.

To achieve this aim, ***the following objectives*** are set:

– to develop a mathematical framework enabling the study of tool wear in the *DCS* as a unified interconnected system, whose evolution is driven by *IET* in the cutting zone;

– to account, in the development of the mathematical model, for the fact that the wear rate depends not only on the current power of *IET* but also on its history, and is influenced by all *DCS* parameters and technological regimes;

– to establish a methodology for the experimental-analytical study of evolutionary changes in the *DCS*, including methods for parameterization of the evolutionary dynamics equations;

– to provide an example of its application in longitudinal turning of *A508-3* steel.

The materials presented in the paper characterize the applications of the synergetic method of analysis and synthesis to the problems of machining on metal-cutting machine tools [56–58].

Methods

First, we turn to the mathematical modeling of the evolutionary *DCS*, in which we take into account the functional relationship of the work ***A(t)*** and power ***N(t)*** with the parameters of the dynamic interaction formed by the cutting process. In turn, based on the study of the relationship between wear and the parameters of the dynamic interaction, changes in the system properties, including the work ***N(t)***, are taken into account as evolutionary changes in wear occur. The evolutionary changes in wear are represented as a function of the "power–work" phase trajectory, for which the *Volterra* integral operator of the second kind is employed [59]. It is also assumed that the workpiece stiffness is an order of magnitude higher than that of the tool subsystem. The evolutionary properties of the *DCS* are then revealed by analyzing the system:

$$
\begin{cases}
\boldsymbol{m}\dfrac{d^2\boldsymbol{X}}{dt^2}+\boldsymbol{h}\dfrac{d\boldsymbol{X}}{dt}+\boldsymbol{cX}=\boldsymbol{F(L,V,X,p)}+\boldsymbol{f(t)}; & (a)\\
\boldsymbol{p(w)}=\{p_1(w),p_2(w),\ldots p_n(w)\}; & (b)\\
N(t)=F^{(0)}(t)V_{\Sigma}(t);\ A(t)=\int\limits_0^t N(\xi)\,d\xi; & (c)\\
T^{(0)}\dfrac{dF^{(0)}}{dt}+F^{(0)}(t)=\rho\left[1+\mu\exp[\varsigma(V_3-v_3)]\right](t_P^{(0)}-X_1)\int\limits_{t-T}^{t}\{V_2-v_2(\xi)\}d\xi, & (d)\\
w(A)=\alpha_1 A+\alpha_2\int\limits_0^{A(t)}\{\exp[-\lambda_1(\quad-\xi)]+\mu_w\exp[\lambda_2(A-\xi)]\}N(\xi)d\xi\ ; & (e)
\end{cases}
\qquad (1)
$$

where ***m***, ***h***, ***c*** are the matrices of inertial, velocity, and elastic coefficients of the tool subsystem, with dimensions [kg·s^2/mm], [kg·s/mm], and [kg/mm], respectively; $\boldsymbol{X}=\{X_1,X_2,X_3\}^T\in\Re_X^{(3)}$, $\boldsymbol{dX/dt}=\boldsymbol{v}=\{v_1,v_2,v_3\}^T\in\Re_X^{(3)}$ are the vectors of tool deformations [mm] and their velocities [mm/s]; $\boldsymbol{F(L,V,X,p)}=\{F_1,F_2,F_3\}^T\in\Re_X^{(3)}$ is the cutting force [kg], expressed in terms of deformations and the trajectories of the machine tool actuating elements *(MTAE)* $\boldsymbol{L}=\{L_1,L_2,L_3\}^T\in\Re_L^{(3)}$ [mm] and their velocities $\boldsymbol{dL/dt}=\boldsymbol{V}=\{V_1,V_2,V_3\}^T\in\Re_L^{(3)}$ [mm/s]; $\boldsymbol{f(t)}=\{f_1,f_2,f_3\}^T\in\Re_X^{(3)}$ is the disturbance vector.

In addition, the cutting force $\boldsymbol{F}=F^{(0)}\{\chi_1,\chi_2,\chi_3\}^T$ can be represented in terms of its magnitude $\boldsymbol{F^{(0)}}$ and angular coefficients $\boldsymbol{\chi_i}$ satisfying the normalization conditions.

System (1) reveals the following mechanisms of mutual influence between the subsystems. Subsystem (1*a*) describes the transformation of forces $\boldsymbol{F(L,V,X,p)}+\boldsymbol{f(t)}$ acting on the tool into deformations ***X***. If the cutting system is autonomous, then $\boldsymbol{f(t)}=0$. Expression (1*b*) states that a functional relationship between wear ***w*** and the parameters $\boldsymbol{p_i}$ of the mathematical model of the dynamic interaction is specified. Dependencies (1*c*) clarify the meaning of the formation of the work and power of irreversible energy transformations. If $V_\Sigma = const$ and $F^{(0)} = const$, then the work differs from time by a constant coefficient. Here, $\boldsymbol{F}^{(0)}$ is the total force acting in the direction of the cutting speed $\boldsymbol{V}_\Sigma$. Moreover, the cutting speed consists of the velocity of the *MTAE* and the elastic deformation velocity. Equation (1*d*) defines the dynamic interaction formed by the cutting process; that is, according to *Haken's* expression [56], it describes the interaction of the system with the environment (the machining process).

This equation is based on the hypothesis that the magnitude of the cutting force is proportional to the cross-sectional area of the cut layer. The proportionality coefficient $\rho[1+\mu\exp[\varsigma(V_3 - v_3)]]$ has the meaning of the chip pressure on the tool rake face [kg/mm^2]. It depends on the cutting speed $V_P^{(0)} = (V_3 - v_3)$. When determining the cross-sectional area of the cut layer $S = (t_P^{(0)} - X_1)\int\limits_{t-T}^{t}\{V_2 - v_2(\xi)\}d\xi$, not only the dependence of the current depth of cut on deformation $t_P(t) = (t_P^{(0)} - X_1)$ is taken into account, but also the variation of the feed rate $S_P^{(0)} = \int\limits_{t-T}^{t}\{V_2 - v_2(\xi)\}d\xi$ as a function of the deformation velocities $\boldsymbol{v_2(t)}$ and $\boldsymbol{v_3(t)}$. All parameters in (1*d*) depend on wear. However, the main parameters affecting stability are $\boldsymbol{p_1} = \rho$ and $\boldsymbol{p_2} = \boldsymbol{T}^{(0)}$.

Finally, equation (1_e) represents a modified *Volterra* integral equation of the second kind [59], which reveals the dependence of wear not on the work of forces, but on the "power–work" trajectory. Moreover, the kernels of the integral evolution operators account for two opposing effects: the adaptation of the tool to the cutting process and its degradation. The parameters of this integral equation, obtained through their identification, have the following dimensions: $\boldsymbol{\lambda_1}\rightarrow|(\text{kg}\cdot\text{m})^{-1}|$ $\boldsymbol{\lambda_2}\rightarrow|(\text{kg}\cdot\text{m})^{-1}|$, $\boldsymbol{\alpha_1}\rightarrow|(\text{kg})^{-1}|$, $\boldsymbol{\alpha_2}\rightarrow|\text{s}/(\text{mm}\cdot(\text{kg})^2|$ and $\boldsymbol{\alpha_3}\rightarrow|\text{s}/(\text{mm}\cdot(\text{kg})^2$.

If system (1) is analyzed as a whole, it should be classified as a system of integro-differential functionally coupled equations [59]. The functional coupling is due to the interdependence of forces on the system parameters, which themselves change as work is performed. Functional coupling gives rise to self-organization [58], which, as can be seen from system (1), depends both on the parameters of the interacting subsystems and on the initial parameters $\boldsymbol{p(w)}$ of the dynamic interaction. Structurally, all coordinates evolve along the work trajectory and remain far from equilibrium, which corresponds to the conditions of dynamic system restructuring [57]. The complexity of digital simulation of system (1) lies primarily in the computation of the *Volterra* integral operator. To compute it, we consider (1*e*) in the following form:

$$W(A) = H_1(A) + H_2(A) + H_3(A), \tag{2}$$

where

$$H_1(A) = \alpha_1 A;$$

$$H_2(A) = \alpha_2\int\limits_0^A \exp\left[-\lambda_1(A-\xi)\right]N(\xi)\,d\xi;$$

$$H_3(A) = \alpha_3\int\limits_0^A \exp\left[\lambda_2(A-\xi)\right]N(\xi)\,d\xi;$$

$$N(A) = \{N(\Delta A), N(2\Delta A), N(3\Delta A), \ldots N(n\Delta A)\};$$

$$A = n\Delta A;$$

$\boldsymbol{\Delta A}$ is the discrete value of the work of forces of irreversible energy transformations of the mechanical system, supplied to the cutting zone; $\boldsymbol{\Delta A}$ is the increment value determined by the *Nyquist* frequency.

The computation of each component of expression (2) then yields

$$\begin{cases} H_1(A) = \alpha_1 n\Delta A; \\ H_2(A) = \dfrac{\alpha_2}{\lambda_1} e^{-\lambda_1 \sum\limits_{i=1}^{n} i\Delta A} \left\{ N_1 e^{-\lambda_1 (n-1)\Delta A)} [1 - e^{-\lambda_1 (\Delta A)}] + N_2 e^{-\lambda_1 (n-2)\Delta A)} [1 - e^{-\lambda_1 (\Delta A)}] + \ldots \right. \\ \left. + N_{n-1} e^{-\lambda_1 \Delta A)} [1 - e^{-\lambda_1 (\Delta A)}] + N_n e^{-\lambda_1 (n-n)\Delta A)} [1 - e^{-\lambda_1 (\Delta A)}] \right\}; \\ H_3(A) = \dfrac{\alpha_3 \left[\exp(-\lambda_2 \Delta A) - 1 \right]}{\lambda_2}. \end{cases} \quad (3)$$

The full discrete analogue of (1*e*) is taken as the basis for modeling the evolutionary changes of the dynamic cutting system:

$$w(A) = \alpha_1 n\Delta A + \frac{\alpha_2 \left[1 - \exp(-\lambda_1 \Delta A)\right]}{\lambda_1} \sum_{i=1}^{i=n} N_i \exp\left[-\lambda_1 (i-1)\Delta A\right] +$$

$$+ \frac{\alpha_3 \left[\exp(-\lambda_2 \Delta A) - 1\right]}{\lambda_2} \sum_{i=1}^{i=n} N_i \exp\left[-\lambda_2 (i-1)\Delta A\right], \quad (4)$$

where $\boldsymbol{N_i}$ is the average power within the $\boldsymbol{i}$-th increment of work.

As can be seen, the value of the integral operator is determined not only by the integrated power of *IET* at the given work instant, but also by its history. The influence of the history depends on the kernel, which decays exponentially for the adaptation component and diverges for the degradation component. Thus, system (1) is structurally unstable. In such a system, according to I. *Prigogine* [57], various attracting sets – which he called dissipative structures – should form. In particular, if $\boldsymbol{N_i} = \boldsymbol{N} =$ const, then

$$w(A) = \alpha_1 A + N \left\{ \frac{\alpha_2}{\lambda_1} [1 - e^{-\lambda_1 A}] + \frac{\alpha_3}{\lambda_2} [e^{\lambda_2 A} - 1] \right\}. \quad (5)$$

As can be seen, the power trajectory of *IET* "colors" the evolution of tool wear with its properties, which depend on the *DCS*. Thus, the evolution of wear, according to model (1), depends, first, on the parameters of the interacting subsystems; second, it changes with variations in the initial parameters of the dynamic interaction formed by the cutting process; and third, it changes with variations in the control parameters. The control parameters in system (1) are the technological regimes. The features of mathematical model (1) described above make it possible to observe and predict changes in the properties of the *DCS* during its operation. Based on this model, it is possible to estimate not only wear but also the trajectories of form-generating motions responsible for the geometric topology of the workpiece surface formed by cutting. It is important to note that each evolutionary wear trajectory depends on the initial *DCS* parameters and disturbances; that is, it is unique, just as the dynamic structure of a particular machine tool is unique.

To illustrate the methodology for analyzing evolutionary changes in wear, we consider an example of longitudinal turning of a shaft made of *A508-3* steel using digital simulation of system (1) with consideration of (4). The choice of this steel grade was determined by the requirements of the enterprise *AEM-Technologies "Atommash"*, under whose assignment the research was carried out. The experimental part of the study was performed on a test rig based on a modernized *1K62* lathe (Fig. 1).

The modernization consisted of the following. First, the asynchronous motors of the *MTAE* were replaced with adjustable frequency drives for spindle rotation and longitudinal and transverse feed rates. The rotational speeds of the servomotors were controlled by a computer. Second, the tool carriage was replaced with an *STD201-1* measuring system manufactured by *National Instruments* (*USA*), which provided measurements of cutting force and integrated temperature (according to the specification, in the frequency range up to 25 kHz) (Fig. 1). Third, vibrations were measured using *A603C01* accelerometers

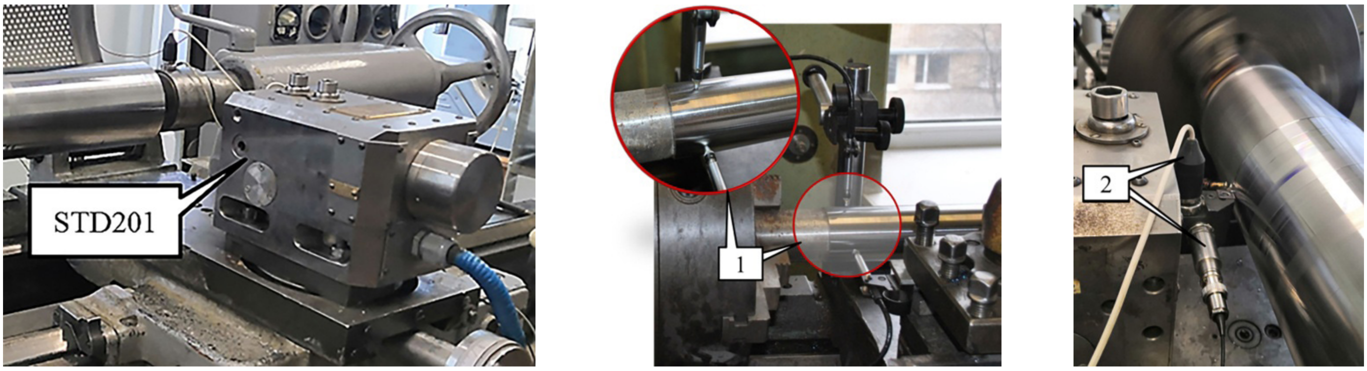


Fig. 1. Experimental unit for studying the evolution of cutting tool wear

with a sensitivity of 10.2 mV/(m/s^2) and a frequency range of 0.4–15,000 Hz, mounted directly on the tool (Fig. 1, item **2**). Non-contact eddy-current displacement sensors were also used to measure the gap between the transducer and the rotating workpiece, which were associated with vibrational sequences of oscillatory displacements (Fig. 1, item **1**). All measuring devices were connected to the computer via a *National Instruments* interface unit (*NI-9234*, *NI-9237*, and *NI-9219* modules) with a sampling frequency of up to 100 kHz. The conversion of accelerations into displacements was performed by digital integration on the computer with trend removal, which was necessitated by the uncertainty of the initial values. Based on the measured data, the values of power and work of cutting forces were also algorithmically computed. Thus, the test rig enables the measurement, during the cutting process, of all information required for estimating the relationship between wear and the main *DCS* coordinates, as well as for parameterization of system (1). When estimating flank wear, the relative value of the force, reduced to a unit length of contact of the cutting edge with the cutting zone, was considered.

The use of adjustable MTAE drives made it possible to prescribe MTAE trajectories using various algorithms from the computer, in particular, according to the algorithm $\boldsymbol{N}$ = const. Then, based on (5), the parameters $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$, $\boldsymbol{\alpha}_3$, $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ of the integral operator (1*e*) can be identified. The identification is based on two experimentally obtained approximations: $\boldsymbol{w(A)}$ and $\boldsymbol{dw(A)/dA = v^{(w)}(A)}$. The identification was performed in two stages. At the first stage, the parameters $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ were estimated from the $\boldsymbol{w(A)}$ curves. Moreover, $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ were estimated at $A \in (0, A^{(min)})$, where $A^{(min)}$ is the value of work at which $v_{(w)}(A)$ = min. This value is easily determined from the $v^{(w)}(A) = \partial w(A)\partial A$ curve. The parameter $\boldsymbol{\lambda}_2$ was estimated at $A \in (A^{(min)}, A^{(max)})$, where $A^{(max)}$ is the work at which wear reaches its critical value. In this study, $w(A^{(max)})$ = 0.8 mm. At the second stage, the parameters $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$, $\boldsymbol{\alpha}_3$ were estimated from the equation (6).

$$\boldsymbol{\alpha A} = \boldsymbol{v}, \tag{6}$$

where $\mathbf{v} = \left\{v^{(w)}(0), v^{(w)}(A^{(\min)}), v^{(w)}(A^{(\max)})\right\}^T$ are the values computed from experimental data; $\alpha = \{\alpha_1, \alpha_2, \alpha_3\}^T$ is the vector of estimated parameters; $\mathbf{A} = \begin{bmatrix} 1 & N & N \\ 1 & N\exp(-\lambda_1 A^{(\min)}) & N\exp(\lambda_2 A^{(\min)}) \\ 1 & N\exp(-\lambda_1 A^{(\max)}) & N\exp(\lambda_2 A^{(\max)}) \end{bmatrix}$.

The methodology for identifying the parameters of the tool subsystem and the dynamic interaction formed by the cutting process is described in detail in our monograph [60, pp. 52–63]. The studies performed on the identification of the *DCS* parameters, as well as the parameters and kernels of the integral operator (1*e*), have revealed the following features.

1. The matrices $\boldsymbol{m}$, $\boldsymbol{h}$, and $\boldsymbol{c}$ remain unchanged during machining on a particular machine tool.

2. Two parameters have the predominant influence on the properties of the dynamic interaction formed by the cutting process. The parameter $\boldsymbol{p_1} = \boldsymbol{\rho}$ depends on the limiting state of the material in the cutting zone, which, for a given tool geometry and cutting speed, is practically constant [38]. Its dependence on cutting speed is modeled by the expression $\rho[1+\boldsymbol{\mu}\exp[V_3 - v_3]]$. Tool wear, which alters the tool geometry

and increases the volume of plastic deformation in the cutting zone, causes an increase in this parameter. The second most significant parameter $\boldsymbol{p_2 = T^{(0)}}$ fundamentally depends on the cutting speed. This is due to the fact that the path traveled by the tool in the direction of the cutting speed, when transitioning from one equilibrium state to another, remains unchanged for a given tool geometry and workpiece material. It increases with wear progression. For the considered machining conditions, the dependence of the parameters $\boldsymbol{\rho(w)}$ and $\boldsymbol{T^{(0)}(w)}$ $\boldsymbol{W}$ can be represented by the following regression equations based on experimental data:

$$\begin{cases} \rho(w) = \rho(0)[1 + \varepsilon_\rho(w)]; \\ T^{(0)}(w) = T^{(0)}(0)[1 + \varepsilon_T(w)]; \end{cases} \quad (7)$$

where

$$\varepsilon_\rho(w) = k^{(\rho)}[\exp(\alpha^{(\rho)} w) - 1];$$

$$\varepsilon_T(w) = k^{(T)}\left[\exp(\alpha^{(T)} w) - 1\right].$$

In the baseline model, the parameters for *A508-3* steel are as follows: $\boldsymbol{k^{(\rho)}} = 3.0$; $\boldsymbol{\alpha^{(\rho)}} = 0.8$ mm; $\boldsymbol{k^{(T)}} = 1.6$; $\boldsymbol{\alpha^{(T)}} = 0.8$ mm^{-1}. Thus, the chip pressure $\boldsymbol{\rho}$ and the chip formation time constant $\boldsymbol{T^{(0)}}$ tend to increase as tool wear progresses.

3. The parameters of the integral equation (1*e*) $\boldsymbol{\alpha_1}$, $\boldsymbol{\alpha_2}$, $\boldsymbol{\alpha_3}$, and $\boldsymbol{\lambda_2}$ remain constant; however, the evolutionary trajectories depend on the *DCS* parameters. Therefore, variations in the initial *DCS* parameters affect all interconnected evolutionary trajectories, including the wear trajectory along the path of the work performed during cutting.

The wear magnitude was estimated by the height of a rectangle whose area was equal to the area of the wear trace on the tool flank face. The wear trace area was measured using a grid on tool microscopes. Physical wear tests were also conducted on the same test rig. Tables 1–3 present an example of the initial parameters of the *DCS* and the integral equation (1*e*) for longitudinal turning of *A508-3* steel under the following conditions: cutting speed $\boldsymbol{V_p} = 1.2$ m/s; feed rate $\boldsymbol{S_p} = 0.12$ mm/rev; cutting depth $\boldsymbol{t_p} = 1$ mm. The cutting conditions were selected as standard for the enterprise *AEM-Technologies "Atommash"*. The reduced mass of the tool subsystem is $\boldsymbol{m} = 4\cdot10^{-3}$ kg·s^2/mm.

Table 1

Parameters of the tool subsystem

$h_{1,1}$ (kg·s/mm)	$h_{2,2}$ (kg·s/mm)	$h_{3,3}$ (kg·s/mm)	$h_{1,2} = h_{2,1}$ (kg·s/mm)	$h_{1,3} = h_{3,1}$ (kg·s/mm)	$h_{2,3} = h_{3,2}$ (kg·s/mm)
2.5	1.3	0.85	0.1	0.08	0.08
$c_{1,1}$ (kg/mm)	$c_{2,2}$ (kg/mm)	$c_{3,3}$ (kg/mm)	$c_{1,2} = c_{2,1}$ (kg/mm)	$c_{1,3} = c_{3,1}$ (kg/mm)	$c_{2,3} = c_{3,2}$ (kg/mm)
10,000	4,000	2,000	2,000	500	400

Table 2

Parameters of the dynamic link when $w(A) = 0$

Material	$\boldsymbol{\rho}$ (kg/mm^2)	$\boldsymbol{\sigma}$ s/mm)	$\boldsymbol{T^{(0)}}$ (s)	$\boldsymbol{\mu}$
A508-3 steel	170	0.4	$1.6\cdot10^{-3}$	0.4

Table 3

Identified parameters of the integral equation

Material	$\boldsymbol{\alpha_1}$ (kg^{-1})	$\boldsymbol{\alpha_2}$ ((s/kg^2·mm)$^{-1}$)	$\boldsymbol{\alpha_3}$ ((s/kg^2·mm)$^{-1}$)	$\boldsymbol{\lambda_1}$ ((kg·mm)$^{-1}$)	$\boldsymbol{\Lambda_2}$ ((kg·mm)$^{-1}$)
A508-3 steel	$0.82\cdot10^{-5}$	$0.5\cdot10^{-5}$	$0.12\cdot10^{-5}$	0.4	0.2

Results and discussion

First, the results of digital simulation of wear evolution are considered. Figs. 2,*a* and 2,*b* show the computed wear curves over time for two initial values of $\boldsymbol{\rho}_{(0)}$. Also, in Fig. 2,*c*, examples of tool deformation trajectories in the X_1-direction are presented.

The diagrams are presented as functions of time, up to the point when flank wear reaches 0.8 mm. Variations in the initial value **ρ(0)** alter the entire evolutionary trajectory. Moreover, along the trajectory ***A(t)***, bifurcations of attracting sets are possible. Fig. 3 shows examples of evolution trajectories for **ρ(0)** = var and $k^{(w)}$ = 0.2. The bifurcation points are marked as **A**, **B**, **C**, **D**, and **E**. These correspond to inflection points on the wear evolution curves. The wear rate, as can be seen from the illustrations, is always positive ($dw/dt > 0$). If the "slowly" shifting equilibrium is stable, the wear development diagrams are shown in Fig. 3 by solid lines up to the bifurcation points. After the bifurcation points, marked as **A**, **B**, **C**, **D**, and **E**, a deviation of the evolutionary trajectories is observed. The bifurcation points on the wear diagram (Fig. 3, *a*) and on the time trajectories (Fig. 3,*b*) coincide. If, during the computation of the evolutionary trajectory in equation (1*a*), we set $d^2X/(dt)^2 = 0$ and $dX/dt = 0$, then the system will be absolutely stable throughout the evolution. In this case, the wear curves continue their evolution as shown by the dashed lines, meaning that the tool wear resistance will be higher than that represented by the solid lines. The solid lines account for the fact that at the marked points, either the stability of equilibrium of elastic deformations is lost, or a transformation of one attracting set into another occurs. For example, at point **E**, a transformation of a limit cycle into a chaotic attractor through a cascade of period-doubling bifurcations is observed. It should be noted that the formation of various attracting sets of deformations in the vicinity of the evolutionary deformation trajectory generally causes an increase in the tool wear rate. However, there are known examples where the excitation of vibrations leads to a reduction in the wear rate

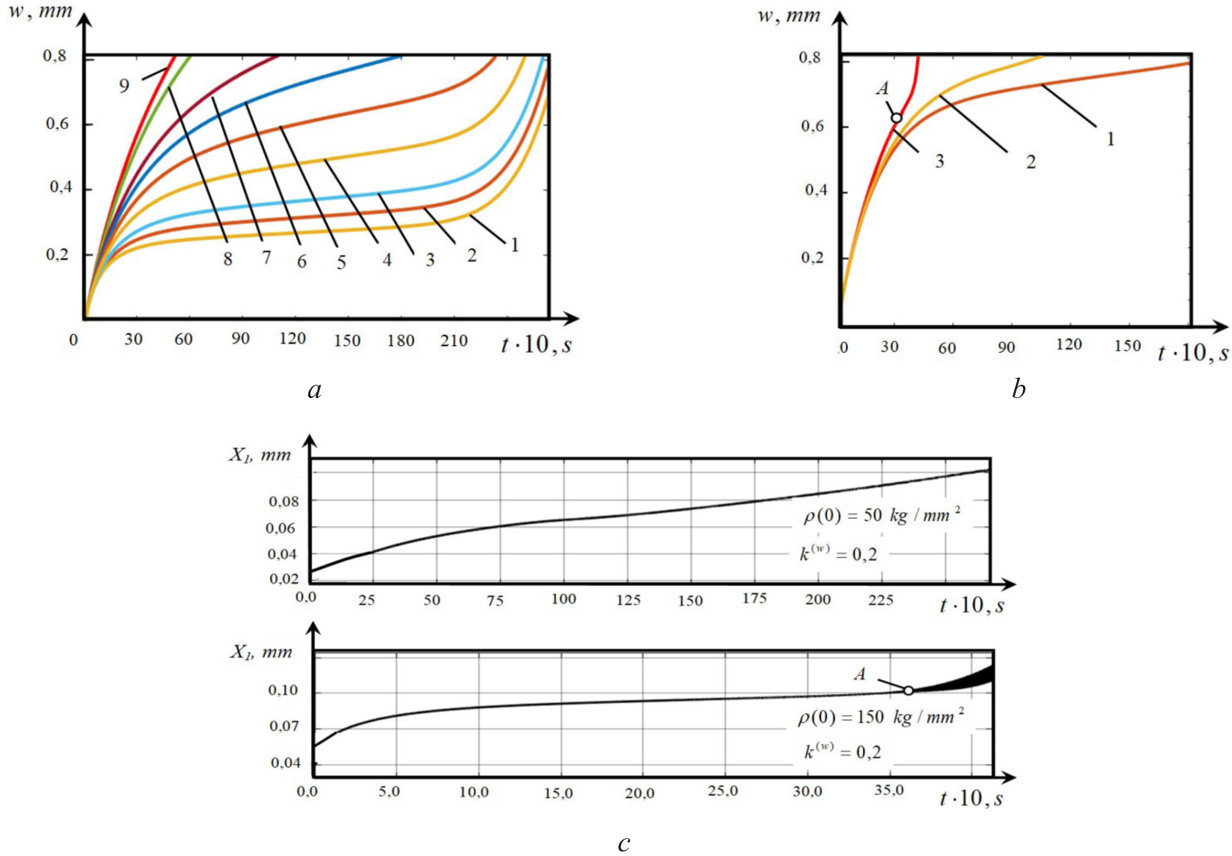
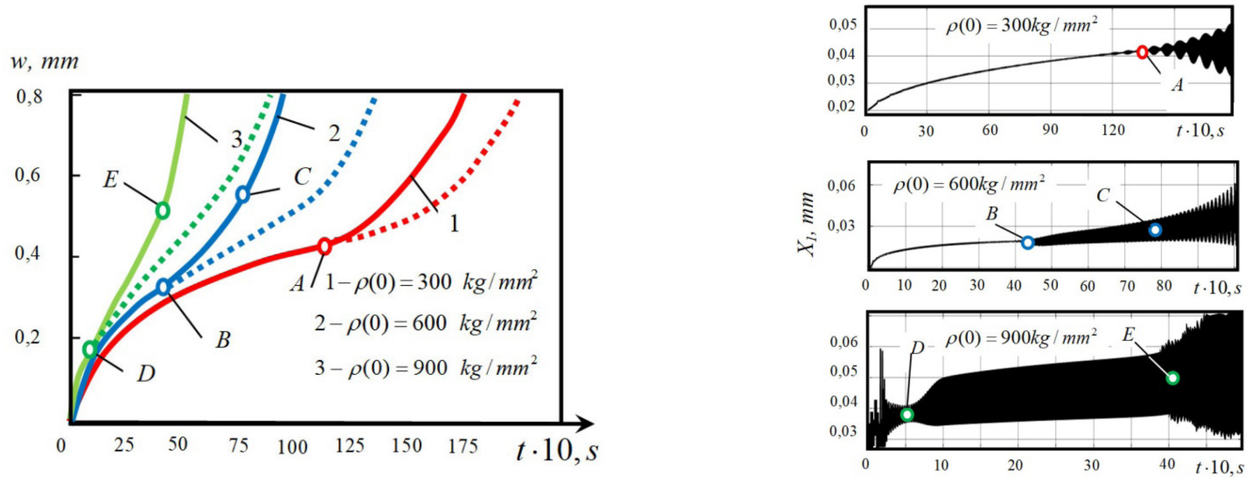


Fig. 2. Diagrams of wear evolution (*a*, *b*) and deformation trajectories (*c*) under variations of **ρ** and $k^{(w)}$:
a – for **ρ** (0) = 50 kg/mm^2 (**1** ⇒ $k^{(w)}$ = 0.05; **2** ⇒ $k^{(w)}$ = 0.1; **3** ⇒ $k^{(w)}$ = 0.2; **4** ⇒ $k^{(w)}$ = 0.3; **5** ⇒ $k^{(w)}$ = 0.4; **6** ⇒ $k^{(w)}$ = 0.5; **7** ⇒ $k^{(w)}$ = 0.6; **8** ⇒ $k^{(w)}$ = 0.7; **9** ⇒ $k^{(w)}$ = 0.8); *b* – for **ρ** (0) = 150 kg/mm^2 (**1** ⇒ $k^{(w)}$ = 0.05; **2** ⇒ $k^{(w)}$ = 0.1; **3** ⇒ $k^{(w)}$ = 0.2)



a *b*

Fig. 3. Evolution of wear (*a*) and deformation displacement trajectories (*b*)

[61]. In such cases, however, restrictions are imposed on the vibration parameters, and compliance with these restrictions is not always achievable.

It is important to note that periodic deformations correspond to almost delta-impulse forces $\boldsymbol{\delta(t–t_i)}$. If the limit cycle is ideal, it corresponds to an ideal pulse sequence of power with a cyclic frequency $\boldsymbol{\Omega^{(3)}} = (\boldsymbol{t_{i+1}–t_i})^{-1}$. This corresponds to a spectrum of forces like delta-impulse $\boldsymbol{\delta(\Omega–\Omega^{(3)}), \Omega\in(0,\infty)}$. The effects of an extreme dependence of the wear rate on time do not always manifest themselves. Depending on the parameters, primarily on the matrices $\boldsymbol{m}$, $\boldsymbol{h}$, $\boldsymbol{c}$ and the parameters of the integral operator, a shift in the minimum of the wear rate occurs.

The sensitivity of the evolution to the energy dissipated in the cutting zone can vary depending on the initial *DCS* parameters. However, the parameters characterizing the dynamic interaction formed by the cutting process are of primary importance. In this regard, we consider the evolutionary diagrams taking into account the simultaneous dependence of the parameters $\boldsymbol{\rho}$ and $\boldsymbol{T^{(0)}}$ on wear (Fig. 4). In Fig. 4, *a*, the points **A**, **B**, **C**, **D**, and **E** mark the moments of bifurcations of attracting sets of deformations. On the deformation trajectories in Fig. 4, *b*, *c*, and *d*, the bifurcation points have the same designations. If the initial parameters of the system are changed in the case shown in Fig. 4, the evolutionary trajectories may change significantly.

A clear illustration of the evolutionary changes is provided by the combined diagrams of the evolution of the parameters ρ and $\boldsymbol{T^{(0)}}$, as well as the figurative line in the plane "$\boldsymbol{\rho(w)} – \boldsymbol{T^{(0)}(w)}$" (Fig. 5). In the illustration, the points at which the system loses stability are marked with five-pointed stars. The arrangement of the figurative lines in the plane of the varying parameters depends both on the invariant parameters of the interacting subsystems and on the evolutionarily changing parameters of the dynamic interaction formed by the cutting process. In particular, in all cases, as the quality factor of the oscillatory circuits formed by the interacting subsystems decreases, the variations in the evolutionary trajectories become less sensitive to changes in the initial system parameters. Digital experiments also show that the evolutionary trajectories change both with variations in the parameters of the interacting subsystems and with changes in the technological regimes, which act as control parameters in the given system.

The results of studying the evolution under variations of the *DCS* parameters show that the evolutionary trajectories, including the change in wear over time, can exhibit high sensitivity to variations in parameters and disturbances acting on the system. Here we find a complete analogy to the problem of the robustness of dynamical systems, first formulated in the 1950s [62]. It must be acknowledged that, in current experimental studies of tool wear, the sensitivity properties of the evolution to small variations in *DCS* parameters are practically not addressed. This, in our opinion, explains the sometimes fundamental changes in wear resistance observed when transferring research results from one machine tool to another. Furthermore, the developed methodology and mathematical framework make it possible to solve the problem of aligning the evolution with the *DCS* parameters. The results of digital simulation also show that the wear rate in the

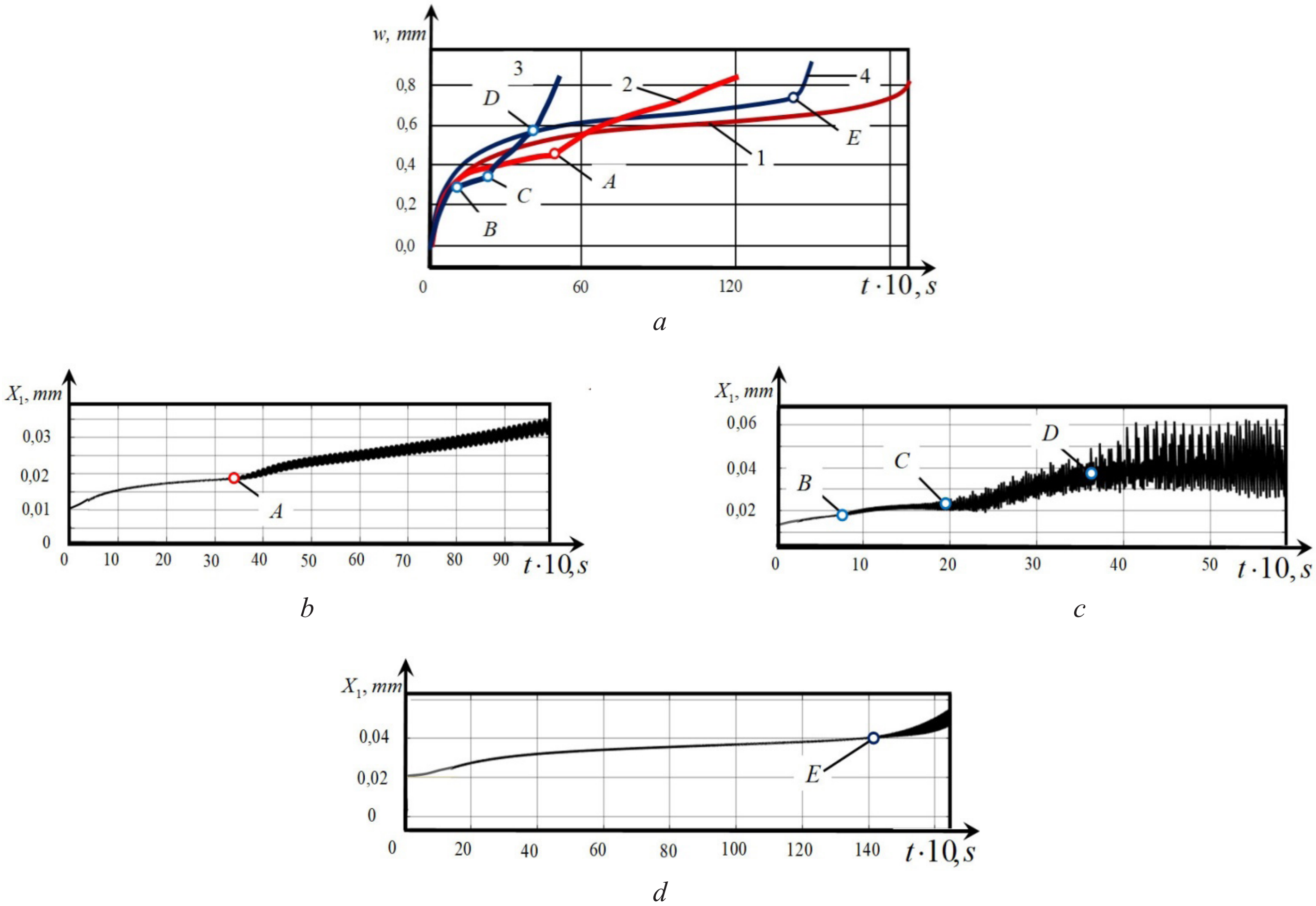


Fig. 4. Evolution of the dynamic cutting system due to the coupled variation of parameters **ρ** and $T^{(0)}$: *a* – wear development diagrams; *b*, *c*, *d* – deformation displacement trajectories in the X_1 direction, corresponding to wear evolution curves **2**, **3**, and **4**, respectively

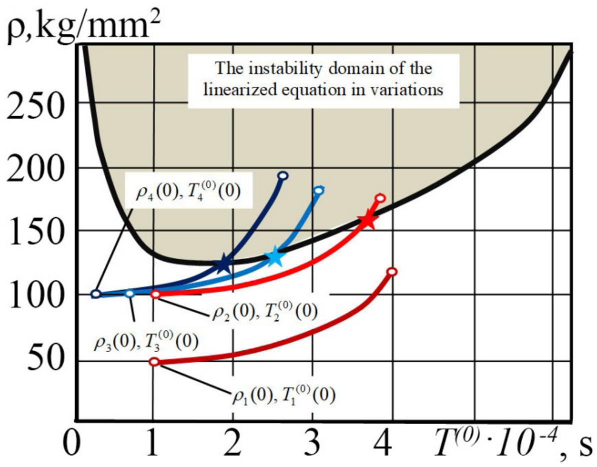


Fig. 5. Superposition of the evolutionary trajectories of the system parameters onto the stability region in the plane of these parameters

same system changes during the course of evolution. The regimes that are optimal from the standpoint of wear resistance also change, primarily the cutting speed. Therefore, the concept of an optimal cutting speed, as commonly used in engineering practice, should be replaced by the concept of an optimal speed trajectory along the work path.

To assess the adequacy of the mathematical framework and the digital simulation results presented above, we consider an illustrative example of experimental determination of wear evolution during longitudinal turning of a shaft made of *A508-3* steel with a tool equipped with 15% *TiC*–6% *Co* carbide inserts. Tool geometry: rake angle γ = 6°, clearance angle α = 6°, principal cutting edge angle φ = 95°, tool nose radius r = 0.5 mm (Fig. 6). It has been shown that the evolution depends on variations in the parameter $\rho_{(0)}$. In experiments, it is more convenient to consider changes in $t^{(0)}_p$, which, together with $\rho_{(0)}$, affects the gain of the *DCS*. However, $t^{(0)}_p$ remains unchanged during the evolution, and only the parameters of the dynamic interaction change, while their initial values remain unchanged.

Each experimental point in the graphs of Fig. 6 corresponds to at least five measurements; the mathematical expectations are shown by circular markers, and variance estimates for the mathematical expectations are also provided. The left panels show the wear curves as the depth of cut $t^{(0)}_p$ increases. The right panels present graphs revealing the dependence of the cutting path L at which wear reaches its critical value.

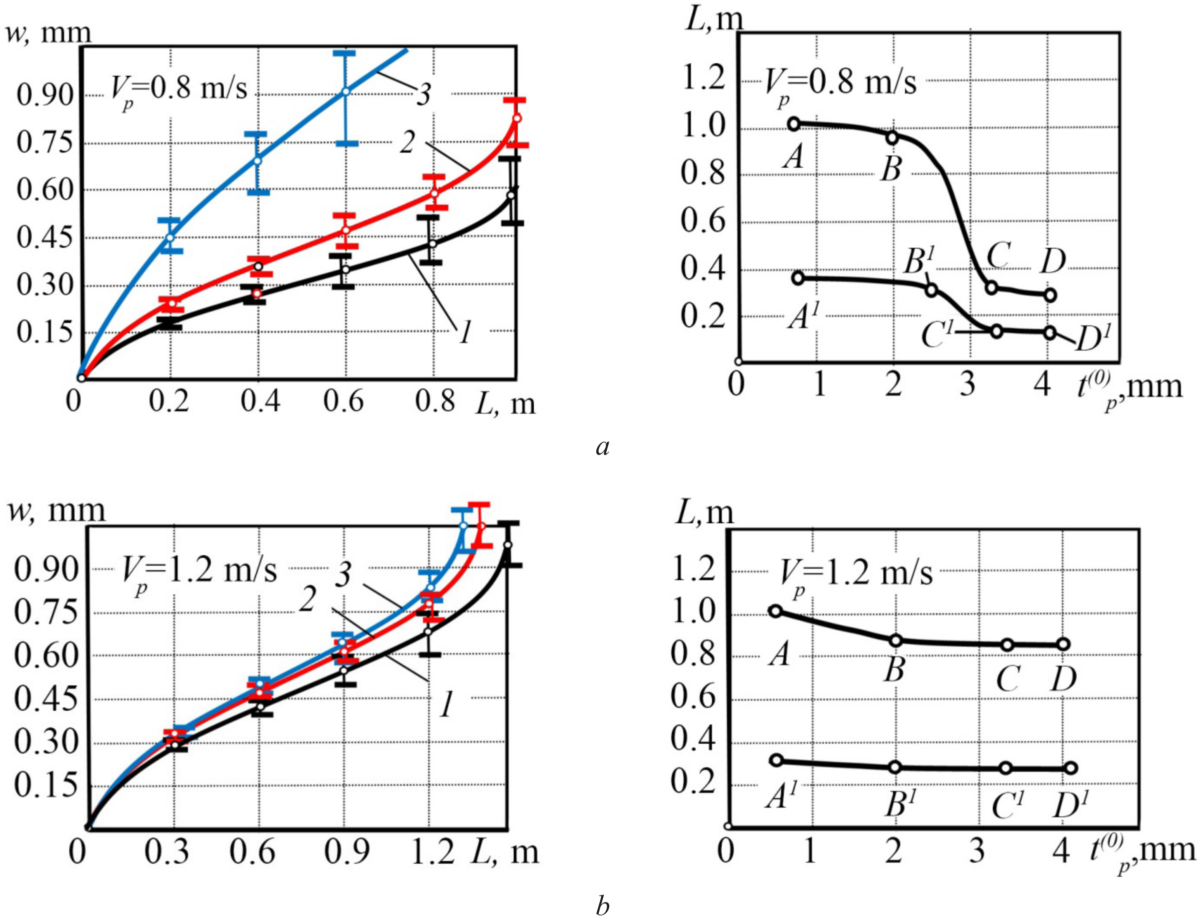


Fig. 6. Variation of wear curves as a function of cutting speeds (*a*) V_p = 0.8 m/s and (*b*) V_p = 1.2 m/s: curves **1** – for a cutting depth of 1.0 mm; **2** – for a cutting depth of 2.0 mm; **3** – for a cutting depth of 3.0 mm

It is noteworthy, first of all, that the wear exhibits a non-monotonic dependence on increasing depth of cut, as well as a dependence on cutting speed. At a cutting speed of 0.8 m/s, variations in depth of cut within the range of 0.5–2.5 mm cause virtually no change in the wear diagram; however, a relatively small further increase in depth of cut results in a several-fold increase in wear increment for the same work increment. This example alone demonstrates that by aligning the depth of cut with the *DCS* parameters, tool life can be increased almost threefold at its limiting value of 0.6 mm. An increase in depth of cut at a cutting speed of 1.2 m/s causes a slight change in the wear rate, with a modest reduction observed. A similar non-monotonic dependence of wear is observed with variations in the tool stiffness matrices, when turning with tools having different overhang from the clamping device. These data confirm that the parameters of the dynamic interaction formed by the cutting process, as well as the stiffness, dissipation, and inertial coefficient matrices of the tool and workpiece subsystems, directly affect the wear rate.

It should be recalled, for example, that in [39, pp. 211–219] it is stated that variations in depth of cut do not affect the wear rate. In the example shown in Fig. 6, turning with a depth of cut of 2.5 mm leads to a loss of stability of the equilibrium of elastic deformations and to bifurcations of their attracting sets as work is performed, following a scenario similar to that in Fig. 4,*c*. Consequently, even a small increase in depth of cut from 2.2 mm to 2.7 mm results in an almost threefold increase in wear rate. It should be emphasized that for a workpiece dynamic subsystem with different stiffness and dissipation matrices, the situation may be different. The weak influence of depth of cut variations on the evolutionary trajectory at a cutting speed of 1.2 m/s is also associated with a decrease in the chip formation time constant as cutting speed increases.

Attention should also be drawn to the effects associated with changes in the quality factor of certain oscillatory circuits formed jointly by the interacting subsystems and the dynamic interaction formed by the cutting process. As a rule, the roots of the characteristic polynomial of the linearized equation in variations about equilibrium move toward the imaginary axis as wear progresses. The quality factor of the oscillatory circuit corresponding to these roots then increases, meaning that the force noise, which always accompanies machining, is amplified in the vibrations [31–35]. In this case, as a rule, when stability is lost as tool wear approaches its critical value, chaotic dynamics develop in the *DCS*. All of these effects are clearly observable when analyzing the evolutionary deformation trajectories in digital simulation of the system (Fig. 3–5).

The developed mathematical model and the presented methodology for identifying its parameters make it possible to study the evolution of cutting tool wear as a unified system that includes all subsystems determining the dynamics of the cutting process as a whole. The presented results of digital simulation show that along the work trajectory of irreversible energy transformations, due to changes in the *DCS* parameters, both slow changes in system properties and bifurcations of attracting sets of deformations and the corresponding forces and, consequently, power formed in the vicinity of the trajectory are observed. In some cases, pitchfork bifurcations occur, and after the bifurcation points, branching of the evolutionary trajectories is observed. In such cases, accurate prediction of wear evolution becomes impossible.

The dependence of *DCS* evolution on parameter variations and disturbances makes it possible not only to align wear evolution with the *DCS* parameters, but also to substantiate the requirements for experimental studies of wear resistance. Such studies should be performed over a range of *DCS* parameters that ensures low sensitivity of the evolution to variations in these parameters. Strictly speaking, experiments must ensure the identity of the dynamic properties of the experimental equipment on which the studies are conducted and the equipment on which the machining will be performed. The developed mathematical framework makes it possible, at the stage of technological production planning, to analyze various manufacturing scenarios on a computer, taking into account the evolutionary changes in machining properties. Furthermore, the mathematical models developed can serve as the foundation for creating a subsystem of evolutionary changes in the cutting process within virtual digital twins of machining.

Conclusion

The developed parameterization methodologies and mathematical models of the evolutionary system have been illustrated using the example of turning *A508-3* steel. The proposed methodologies and the mathematical framework have been implemented at the enterprise *AEM-Technologies “Atommash”* in the development of programs for *CNC* systems.

1. The proposed mathematical framework, along with the methodology for analysis and parameterization of the evolutionary *DCS*, has made it possible to elucidate the main features of the changes in the properties of the cutting process during evolution driven by irreversible energy transformations in the machining zone. Using the example of cutting tool wear evolution, it has been shown that the wear rate is influenced by both the invariant parameters of the *DCS* and the evolutionarily changing parameters of the dynamic interaction formed by the cutting process. The wear trajectory itself, as a rule, exhibits high sensitivity to small variations in the initial *DCS* parameters. In this regard, one of the important characteristics of the machining process is the sensitivity of the evolution to variations in the initial *DCS* parameters and disturbances. On this basis, a direction for improving tool wear resistance has been formulated, which consists in the synergetic alignment of the initial *DCS* parameters with its evolutionary properties.

2. The functional coupling of forces, power, trajectories, and parameters of the dynamic interaction formed by the cutting process has a fundamental influence on the wear rate. These elements form a unified interconnected self-organizing cutting system, in which the control parameters are the technological regimes. Therefore, the developed mathematical framework, combined with the methodologies for parameterizing the equations of evolutionary dynamics, characterizes a virtual digital model of evolution. Its application is promising both at the stage of *CNC* trajectory programming and in the development of systems for estimating and predicting tool wear progression directly during the cutting process using dynamic monitoring systems.

3. It is precisely this mutual influence and the power of irreversible energy transformations dependent on it that lead to the fact that the optimal conditions – for example, from the standpoint of tool wear resistance – are determined not by fixed values of technological regimes, but by their trajectories, which vary along the tool path. The functional interconnectedness of all trajectories of the cutting system leads to a mutual dependence between the evolution of wear and the trajectories of form-generating motions, which are responsible for the geometric topology of the surface formed by the cutting process.

The developed parameterization methodologies and mathematical models of the evolutionary system have been illustrated using the example of turning *A508-3* steel. These methodologies and the mathematical framework have been implemented at the enterprise *AEM-Technologies "Atommash"* in the development of programs for *CNC* systems.

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Conflicts of Interest

The authors declare no conflict of interest.