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A locally adaptive wavelet filtering algorithm for images*

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The algorithms based on the decomposition of a noisy image in an orthogonal basis of wavelet functions have been widely used to filter images (especially contrasting ones) over the past four decades. In this case, most wavelet filtering algorithms are of a threshold nature, namely: the decomposition coefficient smaller in an absolute value of a certain threshold value is reset to zero; otherwise the coefficient undergoes some (most often nonlinear) transformation. A certain (and very significant) drawback of threshold algorithms is that all coefficients of a certain decomposition level are processed with one identical threshold value (i.e., a constant value for all de-composition coefficients). This does not allow taking into account the “individual energy” of each decomposition coefficient for its more optimal processing. Therefore, we propose its own filtering factor for each coefficient, built on the basis of the optimal Wiener filtering and where a filtering parameter is introduced to compensate for incomplete a priori information on the value of the processed decomposition coefficients. In order to select a filtering parameter, a statistical approach has been proposed that makes it possible to estimate the optimal value of this parameter with acceptable accuracy. The performed computational experiment has shown the developed algorithm effectiveness for wavelet filtering of images.

Keywords: wavelet functions, two-dimensional wavelet functions, wavelet image filtering algorithms, wavelet filtering errors, filtering factors, optimal filtering factor, quasi-optimal filtering factor, selection of the optimal filtering parameter

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1. INTRODUCTION AND RESEARCH OBJECTIVES

In the last two decades, the algorithms, based on the representation of the filtered image in the basis of wavelet functions are often used for image filtering [1, 2]. An overview of the wavelet functions used for this can be found in [3]. Algorithms include three main stages [4, 5]:

1) calculation of direct discrete wavelet transform (finding the decomposition coefficients for noisy image values);
2) processing of “noisy” decomposition coefficients;
3) calculation of the inverse discrete wavelet transform from the processed decomposition coefficients (finding the “filtered” image values).

The combination of these three stages is called *wavelet filtering*. Obviously, the quality of filtering a noisy image will depend both on the chosen decomposition basis and on the decomposition coefficient processing algorithm, used at the second stage. But it should be noted that the second factor plays a predominant role.

In *threshold algorithms* (which have become widespread in practice), the decomposition coefficient, which is less in absolute value than a certain threshold value, vanishes; otherwise, such a coefficient is preserved or undergoes some (in the general case, nonlinear) transformation. In foreign recourses, such processing is mentioned as *thresholding*. The threshold value is a kind of "control" parameter, which value significantly depends on the filtering error (for more details, see [6, 7]). An overview of the threshold functions used is given in [8]. In this case, the threshold value can be set by one value for the coefficients of all decomposition levels (*independent threshold level*) or for each decomposition level by a separate one (*dependent threshold level*). Looking ahead, we note that in practice, “hard” and “soft” threshold functions have become widespread which have very significant drawbacks. The most important thing is that all coefficients of a certain decomposition level are processed with one identical threshold value [4, 5]. This does not allow taking into account the “individual energy” of each decomposition coefficient during its thresholding and does not provide the possibility of obtaining minimal filtering errors. In addition, the optimal value estimation of the threshold value (minimizing the filtering error) is a very difficult task, in practice (for a review of various algorithms for choosing a threshold, see [4, 7]).

An essential feature of *multiplicative algorithms* is the selection of an individual multiplier for each noisy decomposition coefficient (in foreign recourses, such processing is called *shrinkaging*). An example is the *Wiener wavelet filtering algorithm*, where the multiplier (varying in the interval) for each coefficient is determined from the condition of the minimum mean square error in estimating each decomposition coefficient, which guarantees a minimum root mean square error in filtering the entire image (for more details, see [4, 9]). Unfortunately, the calculation of such optimal multipliers requires setting the decomposition coefficients of the «exact» (not noisy) image, but such information are absent, filtering real images. In [4, 9], several quasi-optimal algorithms are proposed, which are an adaptation of the Wiener filtering algorithm for filtering real images in practice, when the exact image is not known.

A class of wavelet filtering algorithms was proposed in a number of works (for example, see [10–12]), which occupy an intermediate place between the threshold and Wiener algorithms (in the foreign literature they are called *neighshrinking*) due to the decomposition coefficients processing. These algorithms, more or less, take into account the energy of nearby coefficients, processing a noisy decomposition coefficient. However, the proposed expressions for calculating the filtering factor contain values, which definitions are of an intuitive nature; it does not allow obtaining the minimum error in image filtering [12, 13]. The parameter is introduced in [14], which choice allows approaching the minimum filtering error to a certain extent, in order to minimize the filtering error.

In our research paper, we solve the problem of constructing a wavelet filtering algorithm, where there is a filtering factor that changes in the interval [0,1] for each decomposition coefficient. Filtering parameter is introduced to compensate for incomplete a priori information about the value of the processed decomposition coefficients. In order to select this parameter, a statistical approach has been proposed, which makes it possible to estimate the optimal (in terms of the minimum mean square error) value of the filtering parameter with acceptable accuracy. The performed computational experiment has shown the effectiveness of the developed locally adaptive algorithm for wavelet filtering of images.

2. ADAPTIVE WAVELET FILTERING ALGORITHM

Any image can be interpreted as a function of two variables $f(x,y)$. Let us define the basic functions for the wavelet decomposition of such a function. Traditionally in the scientific recourses, a scalable function (paternal wavelet) is denoted as $\varphi(x)$, but $\psi(x)$ – wavelet (mother wavelet). Using the operations of scaling and shifting, orthonormal basis functions are formed from these ones $\{\varphi_{j,n}(x)\}$, $\{\psi_{j,n}(x)\}$ in the space of one variable functions $f(x)$ [15, 16]. Tensor product of functions $\{\varphi_{j,n}(x)\}$, $\{\psi_{j,n}(x)\}$ generates the following basis functions for the decomposition of two variables functions:

$$\begin{gathered}\{\varphi\varphi_{j,n,m}(x,y)=\varphi_{j,n}(x)\varphi_{j,m}(y)\};\ \{\varphi\psi_{j,n,m}(x,y)=\varphi_{j,n}(x)\psi_{j,m}(y)\};\\ \{\psi\varphi_{j,n,m}(x,y)=\psi_{j,n}(x)\cdot\varphi_{j,m}(y)\};\ \{\psi\psi_{j,m,n}(x,y)=\psi_{j,n}(x)\psi_{j,m}(y)\}.\end{gathered} \tag{1}$$

The corresponding decomposition coefficients are usually called as follows [4]:

- approximating coefficients A_j are obtained as the decomposition coefficients in the basis $\{\varphi\varphi_{j,n,m}(x,y)\}$;
- horizontal detailing factors H_j are obtained as the decomposition coefficients in the basis $\{\varphi\psi_{j,n,m}(x,y)\}$;
- vertical detailing factors V_j are obtained as the decomposition coefficients in the basis $\{\psi\varphi_{j,n,m}(x,y)\}$;
- diagonal detailing coefficients D_j are obtained as the decomposition coefficients in the basis $\{\psi\psi_{j,n,m}(x,y)\}$.

In practice, the image is set by the matrix F the size of $N_X \times N_Y$ (decomposition level j_0). At the first level of decomposition (number j_0+1) approximating coefficients are calculated $A_1 = \left\{aa_{j_0+1,n,m}\right\}$, detailing coefficients $H_1 = \left\{ad_{j_0+1,n,m}\right\}$, $V_1 = \left\{da_{j_0+1,n,m}\right\}$, $D_1 = \left\{dd_{j_0+1,n,m}\right\}$ $D_1 = \left\{dd_{j_0+1,k,n}\right\}$. At the second level of decomposition (number j_0+2) the matrix of coefficients is subjected to the similar processing $A_1 \to (A_2, H_2, V_2, D_2)$. Summarizing the data, we come to the following decomposition scheme:

$$F \to (A_1, H_1, V_1, D_1) \to (A_2, H_2, V_2, D_2, H_1, V_1, D_1) \to \ldots$$

Let us note the regularity of changing two-dimensional arrays sizes of decomposition coefficients, namely: at each decomposition level, the sizes of new coefficients arrays are halved compared to the previous arrays. In this case, the sum of the of coefficients arrays size is equal to the size of the original matrix *F*, which indicates the "volume" preservation of information contained in *F*.

For example, let the original matrix F has sizes $N_X = 512$ (number of rows) × × $N_Y = 384$ (number of columns). Then the arrays of coefficients A_1, H_1, V_1, D_1 have sizes 256×192, arrays of coefficients A_2, H_2, V_2, D_2 – have sizes 128×96 so on.

The inverse two-dimensional wavelet transform is carried out according to the scheme:

$$\ldots \to (A_2, H_2, V_2, D_2, H_1, V_1, D_1) \to (A_1, H_1, V_1, D_1) \to F.$$

According to the level j by $j-1$ all matrices sizes of the coefficients are doubled.

We suppose the registered image values $f(x,y)$ are represented by matrix $\tilde{F}$ by the size $N_X \times N_Y$, which elements can be represented:

$$\tilde{F}_{i_X,i_Y} = F_{i_X,i_Y} + \eta_{i_X,i_Y}, \tag{2}$$

where F_{i_X,i_Y} – “exact” image values, η_{i_X,i_Y} – random measurement noise with zero mean and variance σ_η^2 and values η_{i_X,i_Y} not correlated with each other. It was shown (see, for example, [4,9]) that in the wavelet decomposition of the matrix $\tilde{F}$ errors in calculating detailing coefficients H_j, V_j, D_j by 2 order and more than the approximating coefficients A_j. Therefore, only these detailing coefficients are processed at the second stage of wavelet filtering. For the convenience of further recording, any of the detailing coefficients will be denoted as $\tilde{d}_{n,m}$, where the indices n, m determine the row and column numbers of the corresponding matrix of decomposition coefficients (the number of the decomposition level is going down).

We should suppose that matrix, used for the wavelet decomposition, $\tilde{F}$ basis functions (1) are orthonormal (see [4, 15, 16] for more details). Then it is proved [4, 9] that the expansion coefficients are random variables and:

- one can take the representation $\tilde{d}_{n,m} = d_{n,m} + \varepsilon_{n,m}$, where $d_{n,m}$ – exact image decomposition coefficients, $\varepsilon_{n,m}$ – random coefficient calculation error, conditioned by noise-induced image measurement;
- have a mathematical expectation $M\left[\tilde{d}_{n,m}\right] = d_{n,m}$ and variance $D\left[\tilde{d}_{n,m}\right] = D\left[\varepsilon_{n,m}\right] = \sigma_\eta^2$;
- errors $\varepsilon_{n,m}$ coefficient calculations are not correlated with each other.

There is a question: how the random decomposition coefficients should be processed $\tilde{d}_{n,m}$ or how to build an estimate $\hat{d}_{n,m}$ for the exact coefficient $d_{n,m}$, in order to "filter out" the error as much as possible $\varepsilon_{n,m}$ and distort the coefficient itself as little as possible $d_{n,m}$, or get the minimum systematic error? In order to answer this question let us consider the mean square error (MSE) of the estimation, which we define by the expression as a criterion characterizing the total estimation error:

$$\Delta(\hat{F}) = M\left[\sum_j \sum_n \sum_m (\hat{d}_{n,m,j} - d_{n,m,j})^2\right], \tag{3}$$

where j – decomposition level. It was proved [4, 9], that the minimum MSE is achieved if the estimate $\hat{d}_{n,m}$ is defined by the expression:

$$\hat{d}_{\mathrm{opt}_{n,m}} = w_{\mathrm{opt}_{n,m}} \tilde{d}_{n,m}, \tag{4}$$

where the optimal filtering factor $w_{\mathrm{opt}_{n,m}}$ has a form:

$$w_{\mathrm{opt}_{n,m}} = \frac{1}{\left(1 + \dfrac{\sigma_\eta^2}{d_{n,m}^2}\right)}. \tag{5}$$

In this case, MSE of the coefficient $d_{n,m}$ is defined as:

$$\Delta\left(\hat{d}_{\mathrm{opt}_{n,m}}\right) = M\left[\left(\hat{d}_{\mathrm{opt}_{n,m}} - d_{n,m}\right)^2\right] = \frac{\sigma_\eta^2}{d_{n,m}^2 + \sigma_\eta^2} d_{n,m}^2. \tag{6}$$

Unfortunately, the constructed optimal multiplier (5) includes the ratio "noise / signal" $S_{\mathrm{opt}_{n,m}} = \dfrac{\sigma_\eta^2}{d_{n,m}^2}$, which is unknown due to ignorance of the values of "exact" decomposition coefficient $d_{n,m}$, is in the denominator. Expressions (4), (5) are of theoretical interest, since they indicate *what to strive for*, constructing estimates that are implemented in practice. One of such estimates, based on iterative refinement of the "noise / signal", was constructed in [4, 9].

We propose another approach to constructing a quasi-optimal filtering factor, which can be implemented in practice and where $d_{n,m}^2$ is estimated by the nearby noisy expansion coefficients. We define a rectangular aperture $A_{n,m}$ centered at point (n, m) of size $(2L_X+1)(2L_Y+1)$, which contains the matrix elements of the processed coefficients with indices $\{n-L_X \le i_X \le n+L_X;\ m-L_Y \le i_Y \le m+L_Y\}$. Further, for each coefficient $\tilde{d}_{n,m}^2$ we define the value:

$$\nu_{n,m}^2 = \frac{1}{(2L_X+1)(2L_Y+1)} \sum_{i_X, i_Y \in A_{n,m}} \tilde{d}_{i,j}^2, \tag{7}$$

which can be interpreted as a sample estimate for the quantity $d_{n,m}^2$. Then the quasi-optimal estimate for the filtering factor (5) can be written in the form:

$$\hat{w}_{n,m} = \frac{1}{\left(1+\alpha\dfrac{\sigma_\eta^2}{\nu_{n,m}^2}\right)}, \tag{8}$$

and the quasi-optimal estimate itself $\hat{d}_{n,m}$ for $d_{n,m}$ we calculate as:

$$\hat{d}_{n,m} = \hat{w}_{n,m}\tilde{d}_{n,m} = \frac{1}{\left(1+\alpha\dfrac{\sigma_\eta^2}{\nu_{n,m}^2}\right)}\tilde{d}_{n,m}. \tag{9}$$

These two expressions contain (in contrast to (5)) the filtering parameter, introduced to "compensate" the errors in the estimation of the value $d_{n,m}^2$. How should we choose this option? Obviously, it is desirable to take the quantity α_{opt} as such a parameter, minimizing MSE of wavelet filtering (3). Unfortunately, due to ignorance of the expansion coefficients $d_{n,m}$ the exact image cannot be calculated with the exact value α_{opt}. Therefore, we modify the selection algorithm, used to estimate the optimal threshold values in the threshold wavelet filtering algorithms (for more details, see [4, 9]). We introduce the following statistical criterion:

$$\rho_W(\alpha) = \frac{1}{\sigma_\eta^2} \sum_{i_Y=1}^{N_Y} \sum_{i_X=1}^{N_X} \tilde{F}_{i_X,i_Y}\left(\tilde{F}_{i_X,i_Y} - \hat{F}_{i_X,i_Y}(\alpha)\right), \tag{10}$$

where $\hat{F}_{i_X,i_Y}(\alpha)$ is an matrix element of the image, obtained by the inverse wavelet transform of the expansion coefficients (9) for a given filtering parameter α. As an estimate for the optimal filtering parameter α_{opt} the value α_w is taken, at which the random variable $\rho_W(\alpha)$ is in the interval

$$\left[\vartheta_{\frac{\beta}{2},N},\ \vartheta_{1-\frac{\beta}{2},N}\right], \tag{11}$$

where $\vartheta_{\frac{\beta}{2},N}$, $\vartheta_{1-\frac{\beta}{2},N}$ – quintiles χ_N^2 – distributions with the number of freedom degrees $N = N_X N_Y$ levels $\frac{\beta}{2}$, $1-\frac{\beta}{2}$, respectively; β – the probability of the first kind error, testing the statistical hypothesis about the optimality of the parameter α_w (usually $\beta = 0.05$). In the process of filtering images, the value $N > 30$ and therefore in order to calculate the quantiles χ_N^2 – distribution at $\beta = 0.05$ use expressions:

$$\vartheta_{0.025,N} = N - 1.96\sqrt{2N}, \quad \vartheta_{0.975,N} = N + 1.96\sqrt{2N}. \tag{12}$$

We should note that the calculation is reduced to solving the nonlinear equation

$$\rho_W(\alpha) = N. \tag{13}$$

However, the iterative process stops as soon as $\rho_W(\alpha^{(n)})$ is in the interval (11). The number of iterations is much less than, searching for the root of a nonlinear equation with a given accuracy $\varepsilon \in [10^{-8}, 10^{-6}]$. This makes it possible to effectively use "slow" iterative algorithms (for example, the dichotomy method – dividing a segment in half).

Introducing local evaluation $\nu_{n,m}^2$ and choosing a filtering parameter α from the condition of the minimum filtering MSE (i.e., its adaptation to a specific processed image), allows us to call the proposed wavelet filtering algorithm with a filtering factor (8) a *locally adaptive algorithm.*

An essential feature of the computation algorithm is the use of noise variance to compute σ_η^2. In practice, this value is unknown, and in this case, it is possible to use the estimate

$$\hat{\sigma}_\eta^2 = \left[\frac{median(|\tilde{d}_{n,m}|)}{0.6745}\right]^2,$$

where operator $median(|\tilde{d}_{n,m}|)$ calculates the median of the absolute values of diagonal detailing coefficients of the decomposition level $j_0 + 1$.

3. COMPUTATIONAL EXPERIMENT RESULTS

Due to the nonlinear nature of the estimation procedure it is impossible to do analytical studies of the filtering factor (8) and therefore a numerous computational experiment were carried out to filter different (by spectral composition) images. Let us highlight the experiment results with the LENA image (see Fig. 1), which is often used in foreign publications as a test image. In Fig. 1, *a*) an exact image is shown, in Fig. 1, *b*) – noisy image with relative noise level $\delta_\eta = 0.10$, where

$$\delta_\eta = \sqrt{\frac{\sum_{i_X=1}^{N_X} \sum_{i_Y=1}^{N_Y} \left(\tilde{F}_{i_X,i_Y} - F_{i_X,i_Y}\right)^2}{\sum_{i_X=1}^{N_X} \sum_{i_Y=1}^{N_Y} \left(F_{i_X,i_Y}\right)^2}} . \quad (14)$$

a *b*

Fig. 1. Accurate and noisy images

One can see a significant distortion of the exact image by normally distributed measurement noise. On fig. 2 shows the dependence on the smoothing parameter α :

- relative smoothing error (solid curve)

$$\delta_F(\alpha) = \sqrt{\frac{\sum_{i_X=1}^{N_X} \sum_{i_Y=1}^{N_Y} \left(\hat{F}_{i_X,i_Y}(\alpha) - F_{i_X,i_Y}\right)^2}{\sum_{i_X=1}^{N_X} \sum_{i_Y=1}^{N_Y} \left(F_{i_X,i_Y}\right)^2}} ; \quad (15)$$

- statistics $\rho_W(\alpha)$ – a dotted curve is shown in the figure;
- quintiles $\vartheta_{0.025,N}$, $\vartheta_{0.975,N}$ – are shown by dashed lines.

For the convenience of display in the figure, the last three quantities are divided by the value $N = N_X N_Y = 65536$. As it follows from (11), as the filtering parameter values are accepted for which the statistics values are between dashed lines (quintiles (12)), which, have become one dashed line due to the scale of the figure. Therefore, as an estimate α_W the value is taken α, for which the function $\rho_W(\alpha)$ is between these dashed lines.

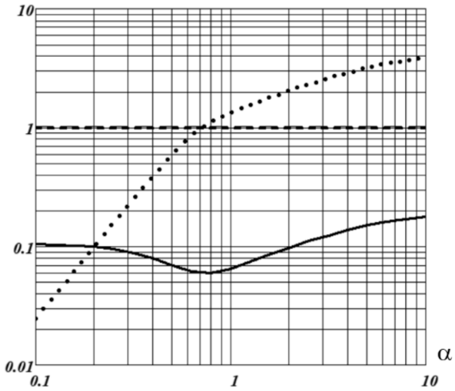

Fig. 2. Characteristics algorithm of filtration

Fig. 3. Filtered image of LENA

An analysis of these graphs allows us to conclude that the proposed approach to choosing a filtering parameter allows us to calculate values α_W from the region of the minimum of the relative filtering error. In this experiment, at $\delta_\eta = 0.10$, the relative filtering error $\delta_F(\alpha_W)$ is 0.046. The filtered image is shown in fig. 3. For a relative noise level of 0.15 – $\delta_F(\alpha_W)$ equals 0.061, which indicates good stability of the proposed filtering algorithm to measurement noise.

CONCLUSION

The proposed locally adaptive filtering algorithm is essentially an adaptation of the optimal Wiener algorithm to the real information available in this experiment. The introduced filtering parameter and its choice from the condition of the minimum MSE makes it possible to compensate for the lack of a priori information on the expansion coefficients of the exact image. The comparison with the filtering results, the same noisy images by threshold algorithms (for more details see [4, 7]) shows that the proposed algorithm has 15…20 % less filtering MSE, although it requires more operations due to the need to calculate the values (7).

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*Локально-адаптивный алгоритм вейвлет-фильтрации изображений**

Ю.Е. ВОСКОБОЙНИКОВ

Аннотация

На протяжении четырех последних десятилетий для фильтрации изображений (особенно контрастных) широко используются алгоритмы, основанные на разложении зашумленного изображения в ортогональном базисе вейвлет-функций. При этом большинство

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алгоритмов вейвлет-фильтрации носят пороговый характер, а именно: коэффициент разложения, меньший по абсолютной величине, некоторой пороговой величины зануляется, в противном случае коэффициент подвергается некоторому (чаще всего нелинейному) преобразованию. Определенным (и весьма существенным) недостатком пороговых алгоритмов является то, что все коэффициенты определенного уровня разложения обрабатываются с одной одинаковой пороговой величиной (т. е. постоянной величиной для всех коэффициентов разложения). Это не позволяет учитывать «индивидуальную энергию» каждого коэффициента разложения для более оптимальной его обработки. Поэтому в настоящей работе предлагается для каждого коэффициента свой фильтрующий множитель, построенный на основе оптимальной винеровской фильтрации и в котором для компенсации неполной исходной информации о величине обрабатываемого коэффициентов разложения вводится параметр фильтрации. Для выбора параметра фильтрации предложен статистический подход, позволяющий с приемлемой точностью оценить оптимальное значение этого параметра. Выполненный вычислительный эксперимент показал эффективность разработанного алгоритма вейвлет-фильтрации изображений.

Ключевые слова: вейвлет-функции, двумерные вейвлет-функции, алгоритмы вейвлет-фильтрации изображений, ошибки вейвлет-фильтрации, фильтрующие множители, оптимальный фильтрующий множитель, квазиоптимальный фильтрующий множитель, выбор оптимального параметра фильтрации

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